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Abstract

摘要

We give an overview of moduli stabilization in compactifications of string theory. We summarize current methods for construction and analysis of vacua with stabilized moduli, and we describe applications to cosmology and particle physics. This is a contribution to the Handbook on Quantum Gravity.

我们概述弦理论紧致化中的模稳定问题，总结当前构造与分析模稳定真空的现有方法，并介绍其在宇宙学和粒子物理中的应用。本文是《量子引力手册》的一篇供稿。

Keywords

关键词

String compactifications - Moduli stabilization - Fluxes - Supersymmetry

弦紧致化 - 模稳定 - 通量 - 超对称

Introduction

引言

The fundamental physical laws that govern our Universe must describe gravity and quantum mechanics. To discover the laws of quantum gravity, we cannot entirely rely on terrestrial experiments, or even on cosmological observations: the energies of observable processes are far too low to give a complete picture, in contrast to the way that collider experiments eventually revealed the Standard Model of particle physics. We may hope for some guidance from experiment, but theorists will have to provide a framework.

支配我们宇宙的基本物理定律必须同时描述引力与量子力学。要发现量子引力定律，我们无法完全依赖地面实验，甚至宇宙学观测也不行: 可观测过程的能量过低，无法呈现完整图景，这和对撞机实验最终揭示粒子物理标准模型的情况全然不同。我们或许可以期待实验提供一些指引，但理论研究者必须给出一套框架。

String theory is such a framework: it is a theory of quantum gravity through which we can take a constructive approach to exploring possible laws of quantum gravity in our Universe. The first obstacle is that the world we observe at low energies is four-dimensional, while the best-understood solutions of string theory are ten-dimensional. Kaluza-Klein theory [1, 2], now more than a century old, provides a way to bridge this gap. If the extra dimensions correspond to a six-dimensional compact space that is smaller than the reach of any experimental probe, then only three spatial dimensions will be seen.

弦理论就是这样一套框架: 它是量子引力理论，我们可以通过它以建构性的方式探索我们宇宙中量子引力的可能定律。第一个难题是，我们在低能下观测到的世界是四维的，但弦理论最广为人知的解是十维的。卡鲁扎-克莱因理论 [1, 2] 诞生至今已有一个多世纪，它提供了弥合这一鸿沟的方法: 如果额外维度对应一个小于任何实验探测分辨率的六维紧致空间, 那么我们就只能观测到三个空间维度。

However, the size and shape of the extra dimensions are dynamical: they are parameterized by the expectation values of scalar fields known as moduli. Unless the moduli have large masses, they mediate long-range forces that are not observed in our world. Thus, a central problem of Kaluza-Klein theories is to provide a dynamical explanation for the requisite size of the extra dimensions, and to ensure that the moduli masses are consistent with experiment. Addressing these challenges is the main obstacle in connecting string theory to observations, and it is the subject of this review. [1]

但额外维度的大小和形状是动力学的: 它们由名为模的标量场的真空期望值参数化。除非模的质量很大，否则它们会产生我们世界中从未观测到的长程力。因此，卡鲁扎-克莱因理论的核心问题是，为额外维度所需的尺寸给出动力学解释，并保证模的质量与实验相符。解决这些难题是将弦理论和观测联系起来的主要障碍，也是本综述的主题。[1]

The Vacuum Problem

真空问题

To understand quantum gravity in our four-dimensional, non-supersymmetric Universe, we will study compactifications of superstring theory on six-dimensional compact spaces, and seek solutions in which supersymmetry is broken. In this section we will carefully explain the reasoning that directs us to the class of solutions that are the subject of this chapter: namely, flux compactifications on orientifolds of Calabi-Yau threefolds.

为了理解我们这个四维非超对称宇宙中的量子引力, 我们将研究超弦理论在六维紧致空间上的紧致化，寻找超对称破缺的解。本节我们将仔细梳理推导思路，引出本章的研究对象: 卡拉比-丘三维型定向对称形上的通量紧致化。

To begin, we take a product ansatz for a ten-dimensional spacetime:

首先，我们给出十维时空的乘积假设:

$$ds^2 = g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(y)\,\mathrm{d}y^m\,\mathrm{d}y^n, \tag{1}$$

for $\mu, v = 0, \ldots, 3$ and $m, n = 4, \ldots, 9$. We suppose that g_{mn} is a Riemannian metric on some compact space ${}^2X_6$. Defining the Ricci tensors $R_{\mu\nu}$ and R_{mn} constructed from $g_{\mu\nu}$ and g_{mn} , respectively, the ansatz (1) solves the ten-dimensional vacuum Einstein equations if and only if $R_{\mu v} = R_{mn} = 0$. Thus, vacuum solutions of string theory are furnished by Ricci-flat six-manifolds.

适用于 $\mu, v = 0, \ldots, 3$ 和 $m, n = 4, \ldots, 9$ 。我们假设 g_{mn} 是紧致空间 ${}^2X_6$ 上的黎曼度量。分别由 $g_{\mu\nu}$ 和 g_{mn} 构造里奇张量 $R_{\mu\nu}$ 和 R_{mn} ，当且仅当 $R_{\mu v} = R_{mn} = 0$ 时，假设 (1) 满足十维真空爱因斯坦方程。因此，弦论的真空解由里奇平坦六流形给出。

Consider a six-manifold X_6 with Kähler metric g_{mn} , and define the Riemannian holonomy group $\mathrm{Hol}(g)$. If $\mathrm{Hol}(g) = SU(3) \subset SO(6)$, then g is Ricci-flat, and we call X_6 a Calabi-Yau threefold. It follows that X_6 having holonomy $SU(3)$ is a sufficient condition for a vacuum solution of string theory in compactification on X_6 . One striking property of such solutions is that the compactification preserves some of the supersymmetry found in the ten-dimensional theory.

考虑一个带有凯勒度量 g_{mn} 的六流形 X_6 ,定义黎曼和乐群 $\mathrm{Hol}(g)$。若满足 $\mathrm{Hol}(g) = SU(3) \subset SO(6)$ ，则 g 是里奇平坦的，我们称 X_6 为卡拉比-丘三维型。由此可得，X_6 具有和乐 $SU(3)$ 是弦论在 X_6 上紧致化得到真空解的充分条件。这类解的一个显著性质是，紧致化会保留十维理论中部分原有的超对称。

We are aware of no argument that $\mathrm{Hol}(g) = SU(3)$ is a necessary condition for Ricci-flatness, but every example of a compact Ricci-flat six-manifold constructed to date has holonomy contained in $SU(3)$ (see, e.g.,[10]). Even so, one should bear in mind the possibility that there may exist compact Ricci-flat six-manifolds with holonomy $SO(6)$ that furnish non-supersymmetric vacua of string theory. By necessity we will restrict our discussion to the known examples of vacuum solutions, i.e., to Calabi-Yau threefolds. We stress, however, that the presence of supersymmetry in this context is not a consequence of an attempt to explain hierarchies in particle physics: it is instead a powerful aid to theoretical control. Specifically, the superstring is our starting point because superstring theories are vastly better-understood than non-supersymmetric string theories, and Calabi-Yau compactifications - which preserve some supersymmetry at the compactification scale, though this may be broken to nothing at lower energies - are our geometric focus because they are the only compact Ricci-flat six-manifolds known at the time of this writing.

目前我们没有论证表明 $\mathrm{Hol}(g) = SU(3)$ 是里奇平坦的必要条件，但迄今为止构造出的所有紧致里奇平坦六流形，其和乐都包含在 $SU(3)$ 中 (例如参见 [10])。即便如此，我们也需要注意，存在具有 $SO(6)$ 和乐的紧致里奇平坦六流形的可能性，这类流形可以给出弦论的非超对称真空。因此我们的讨论必然仅限于已知的真空解，即卡拉比-丘三维型。但我们需要强调，此处出现超对称并不是为了解释粒子物理中的能标层级，它反而只是用来保证理论可控性的有力工具。具体来说，我们以超弦为起点，是因为超弦理论远比非超对称弦论研究得透彻；我们将几何聚焦于卡拉比-丘紧致化——这类紧致化在紧致化能标保留了部分超对称，尽管超对称会在低能下完全破缺——是因为在撰写本文时，它们是仅有的已知紧致里奇平坦六流形。

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[1] For previous reviews on fluxes and the string landscape, see [3-5]. For string cosmology see [6], the recent review [7], and references therein. The geometry of string compactifications is treated in [8]. For a more mathematical perspective on flux compactifications, see [9].

[1] 关于通量和弦景观的早期综述，参见 [3-5]；弦宇宙学相关内容参见 [6]、最新综述 [7] 及其中的参考文献；弦紧致化的几何相关内容参见 [8]；从数学角度研究通量紧致化的内容参见 [9]。

[2] This space may be a proper six-manifold, or it may be some more singular space on which string theory remains well-defined, but in both cases we will use the term ”manifold.”

[2] 该空间可以是正则六流形, 也可以是弦论仍能良好定义的更奇异空间, 两种情况下我们都使用“流形”这一术语。

Vacuum solutions of string theory of the form (1), with $g_{\mu\nu} = \eta_{\mu\nu}$ and with X_6 a Calabi-Yau threefold, are a starting point for discussing cosmology in quantum gravity, but these solutions are far from realistic. The primary shortcomings are two unwanted features: unbroken supersymmetry and light scalar fields known as moduli.

形式如 (1) 的弦论真空解，其中 $g_{\mu\nu} = \eta_{\mu\nu}$ 且 X_6 为卡拉比-丘三维型，是讨论量子引力中宇宙学问题的起点，但这类解远非现实。它们的主要缺陷源于两个不想要的性质: 未破缺的超对称，以及被称为模的轻标量场。

Compactifications of type II string theory on Calabi-Yau threefolds preserve $\mathcal{N} = 2$ supersymmetry in four dimensions, and have exact moduli spaces parameterized by the scalar components of vector multiplets and hypermultiplets. These moduli spaces correspond to geometric deformations of the Ricci-flat metric. Our world exhibits no supersymmetry at all in the infrared, and in particular the chiral representations seen in the Standard Model are incompatible with $\mathcal{N} = 2$ super-symmetry. Moreover, massless scalar fields with gravitational-strength couplings - which is precisely how the geometric moduli manifest in four dimensions - mediate long-range forces [3] that are highly constrained, and are also subject to stringent limits from Big Bang nucleosynthesis. In sum, type II compactifications on Calabi-Yau threefolds, without any sources of stress-energy, are plagued by massless moduli and extended supersymmetry, and so cannot furnish realistic models of our Universe.

II 型弦理论在卡拉比-丘三流行上的紧致化会在四维保留 $\mathcal{N}=2$ 超对称性，其精确模空间由矢量多重态和超多重态的标量分量参数化。这些模空间对应里奇平坦度量的几何形变。我们的世界在红外区域完全不表现出超对称性，尤其是粒子物理标准模型中的手征表示与 $\mathcal{N}=2$ 超对称性不相容。此外，几何模在四维中恰好表现为引力耦合强度的无质量标量场，这类场会传递受到严格限制的长程力 [3]，还会受到大爆炸核合成给出的严苛限制。总而言之，不带任何应力能量源的 II 型卡拉比-丘三流紧致化存在无质量模与 extended 超对称性问题，因此无法给出我们宇宙的现实模型。

More promising are compactifications that are not vacuum solutions, but instead solve the ten-dimensional equations of motion in the presence of sources of stress-energy in the internal space. Quantization of the closed string famously reveals a massless spin-2 excitation, the graviton, but there are other important massless fields, including p -form gauge potentials with associated $(p+1)$ -form field strengths known as fluxes. These potentials and field strengths generalize the one-form potential and the electric and magnetic fields of Maxwell theory, respectively. For example, in type IIB string theory, one finds two-form potentials B_2 and C_2 with associated three-form field strengths $H_3 = \mathrm{d}B_2$ and $F_3 = \mathrm{d}C_2$. Dirac quantization ensures that in an appropriate normalization, these three-form fluxes are elements of $H^3(X_6, \mathbb{Z})$. Quantized fluxes are a critical source of stress-energy, and give the subject of flux compactifications its name. In the type II and type I theories, consistency conditions require that flux appears in association with other, localized sources: D-branes and orientifold planes.

更有前景的是这类紧致化: 它们本身不是真空解，而是在内空间存在应力能量源的情况下满足十维运动方程。众所周知，闭弦量子化会给出无质量自旋 2 激发即引力子，但还存在其他重要的无质量场，包括伴随 $(p+1)$ 形式场强 (称为通量) 的 p 形式规范势。这些势和场强分别是麦克斯韦理论中一形式势以及电场、磁场的推广。例如，在 IIB 型弦理论中存在二形式势 B_2 和 C_2，对应三形式场强 $H_3 = \mathrm{d}B_2$ 和 $F_3 = \mathrm{d}C_2$ 。狄拉克量子化保证了，在适当归一化下，这些三形式通量是 $H^3(X_6, \mathbb{Z})$ 中的元素。量子化通量是关键的应力能量源，通量紧致化也因此得名。在 II 型和 I 型理论中，自洽性条件要求通量必须与其他局域源共存:D 膜和定向对称平面。

In the broadest sense, a flux compactification could mean any compactification of string theory containing p -form fluxes. However, in this chapter we will use a restricted definition that nevertheless encompasses much of the work aiming to describe our Universe in string theory. By a flux compactification, we will mean a compactification of a superstring theory on a Calabi-Yau threefold [4] , or on an orientifold thereof, which includes p -form fluxes and any necessary localized sources. Configurations of this sort are non-vacuum solutions.

广义上，通量紧致化可以指任何包含 p 形式通量的弦理论紧致化。但在本章中，我们会采用一个更狭义的定义，这个定义仍然涵盖了绝大多数旨在用弦理论描述我们宇宙的研究。我们所说的通量紧致化，是指超弦理论在卡拉比-丘三流形 [4] 或其定向对称化流形上的紧致化，其中包含 p 形式通量和所有必要的局域源。这类构型是非真空解。

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[3] These limits apply only to spinless fields that are parity-even: pseudoscalars instead mediate spin-dependent forces that are much less constrained: see section ”Axions”.

[3] 这些限制仅适用于宇称为偶的无自旋场: 赝标量传递的是自旋相关力，这类力的约束宽松得多: 参见“轴子”一节。

The principal motivation for studying flux compactifications is that the stress-energy of fluxes and localized sources ameliorates the problems of massless moduli and unbroken supersymmetry that are inevitable in Calabi-Yau vacuum configurations. A general expectation, which we will explain in detail below, is that flux compactifications that contain sufficiently generic fluxes lead to four-dimensional theories that preserve $\mathcal{N} = 1$ or $\mathcal{N} = 0$ supersymmetry, and in which all moduli are massive. We will refer to the set of such isolated vacua as the landscape of flux vacua [5] .

研究通量紧致化的核心动机是，通量与局域源的应力能量能够缓解卡拉比-丘真空构型中不可避免的无质量模与未破缺超对称性问题。我们下文会详细解释，一个普遍的预期是: 包含足够一般通量的通量紧致化，会得到保留 $\mathcal{N} = 1$ 或 $\mathcal{N} = 0$ 超对称性、且所有模都获得质量的四维理论。我们将所有这类孤立真空的集合称为通量真空景观 [5] 。

In practice it is rarely possible to find vacua that preserve $\mathcal{N} = 0$ supersymmetry and have all moduli massive already at the classical level. Instead one can proceed by finding configurations that preserve $\mathcal{N} = 1$ supersymmetry in some approximation - for example, at leading order in the string loop and α' expansions - and hence have moduli spaces parameterized by the scalar components of chiral multiplets. General arguments predict that the moduli acquire mass from quantum effects and that the exact vacuum configurations are isolated points within the leading-order moduli spaces. Finding these vacua is feasible in some cases, as we will show.

实际研究中，很少能找到在经典水平就保留 $\mathcal{N} = 0$ 超对称性且所有模都获得质量的真空。我们反而可以先找到在一定近似下保留 $\mathcal{N} = 1$ 超对称性的构型——例如在弦圈和 α' 展开的领头阶——这类构型存在由手征多重态标量分量参数化的模空间。一般性论证预测，模会通过量子效应获得质量，精确真空构型是领头阶模空间内的孤立点。我们会在下文说明，在部分情形下找到这些真空是可行的。

To recap, in aiming to describe the Universe within string theory, one seeks isolated and non-supersymmetric solutions of the equations of motion of string theory. A primary strategy of recent decades is to begin with Calabi-Yau vacuum configurations of one of the superstring theories and introduce sources of stress-energy - quantized fluxes, as well as localized sources such as D-branes - that lead to controllably small corrections to the vacuum solution. Among these corrections are supersymmetry-breaking mass splittings, as well as masses for moduli fields.

综上所述，为了在弦理论框架内描述宇宙，我们需要寻找弦理论运动方程的孤立非超对称解。近几十年来的核心策略是: 从某类超弦理论的卡拉比-丘真空构型出发，引入应力能量源——量子化通量，以及 D 膜这类定域源——从而对真空解产生可控的微小修正。这些修正包括超对称破缺的质量劈裂，以及模场的质量。

The study of isolated, non-supersymmetric, non-vacuum solutions is necessarily founded on approximations. No exact solutions are known, and none appears to us to be on the horizon. In this respect, flux compactification falls into the mainstream of theoretical physics, but differs from some areas of string theory, and more general mathematical physics, in which exact methods are prevalent. At the same time, the study of flux vacua owes a great deal to approximation schemes that are founded on systematic expansions around exact results, e.g., using mirror symmetry or nonrenormalization, as we will see.

对孤立非超对称非真空解的研究必然建立在近似方法之上。目前没有已知的精确解，在可预见的未来似乎也不会出现。就此而言，通量紧致化属于理论物理学的主流，它与弦理论某些领域以及更广泛的数学物理学不同，后两者中精确方法占主导地位。同时，正如我们之后会看到的，通量真空研究极大得益于基于精确结果系统展开的近似方案，例如使用镜像对称或非重整化的方法。

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[4] We briefly discuss compactifications of M-theory on manifolds of G_2 holonomy in section ”M-Theory”.

[4] 我们将在“M 理论”一节中简要讨论 M 理论在 G_2 和乐流形上的紧致化。

5 ”The Landscape” is often used to refer to the more general set of all consistent quantum gravity theories, in any number of spacetime dimensions, with or without moduli, but in this chapter we focus on isolated four-dimensional vacua.

5 “景观”通常被用来指代更广泛的所有自洽量子引力理论的集合，涵盖任意时空维数，包含或不包含模都囊括在内，但本章我们聚焦于孤立的四维真空。

Type IIB Flux Compactifications

IIB 型通量紧致化

The primary class of flux compactifications surveyed in this article are compactifi-cations of type IIB string theory on orientifolds of Calabi-Yau threefolds. To explain this class of solutions, we will first introduce the type IIB supergravity action and the two fundamental expansions, in g_s and α' (section ”Scalings and Perturbative Expansions”). In section ”ISD Flux Compactifications” we introduce the central ansatz of imaginary self-dual (ISD) three-form fluxes, and in sections ”Superpoten-tial” and ”Kähler Potential” we examine the superpotential and Kähler potential that describe ISD configurations.

本文研究的主要通量紧致化类别是 IIB 型弦论在卡拉比-丘三维形定向偶上的紧致化。为解释这类解，我们首先会介绍 IIB 型超引力作用量，以及 g_s 和 α' 中的两个基本展开 (见章节“标度与微扰展开”)。在“ISD 通量紧致化”章节，我们介绍虚自对偶 (ISD) 三形式通量的核心假设，在“超势”和“凯勒势”章节，我们研究描述 ISD 构型的超势与凯勒势。

The massless bosonic spectrum of type IIB string theory in ten dimensions consists of the metric g_{MN} , two-form B_2 and dilaton ϕ in the Neveu-Schwarz-Neveu-Schwarz sector, and the p -form potentials C_0, C_2, C_4 in the Ramond-Ramond sector. We define the three-form fluxes:

十维下 IIB 型弦论的无质量玻色子谱包括: 纳维-施瓦茨-纳维-施瓦茨扇区的度规 g_{MN} 、二形式 B_2 和伸缩子 ϕ ，以及拉蒙-拉蒙扇区的 p 形式势 C_0, C_2, C_4 。我们定义三形式通量:

$$F_3 := \mathrm{d}C_2,\ H_3 := \mathrm{d}B_2,\ G_3 := F_3 - \tau H_3, \tag{2}$$

the five-form

五形式

$$\widetilde{F}_5 \ := \ \mathrm{d}C_4 + \frac{1}{2}B_2 \wedge F_3 - \frac{1}{2}C_2 \wedge H_3, \tag{3}$$

and the complex axiodilaton

以及复轴伸缩子

$$\tau \ := \ C_0 + ie^{-\phi}. \tag{4}$$

The full ten-dimensional effective action S for the bosonic fields can be written as $S = S_{10} + S_{\mathrm{loc}}$, with S_{10} a bulk action and S_{loc} encoding the contributions of localized objects such as D-branes. The bulk action $S_{10}^{(0)}$ at leading order in the g_s and α' expansions is

玻色场完整的十维有效作用量 S 可写为 $S = S_{10} + S_{\mathrm{loc}}$ ，其中 S_{10} 是体作用量，S_{loc} 编码 D 膜等定域对象的贡献。领先阶下，g_s 和 α' 展开中的体作用量 $S_{10}^{(0)}$ 为

$$S_{10}^{(0)} = \frac{1}{2\kappa_{10}^2}\int\sqrt{-g}\left(\mathcal{R} - \frac{|\nabla\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{12\,\mathrm{Im}\,\tau} - \frac{|F_5|^2}{4.5!}\right)$$
$$+\frac{1}{8i\kappa_{10}^2}\int\frac{C_4\wedge G_3\wedge\bar{G}_3}{\mathrm{Im}\,\tau} \tag{5}$$

where κ_{10} is the ten-dimensional gravitational coupling and the Einstein-frame metric g_{MN} is related to the string-frame metric $\hat{g}_{MN}$ by $g_{MN} = \sqrt{\mathrm{Im}\,\tau}\hat{g}_{MN}$. The self-duality condition $\widetilde{F}_5 = \star_{10}\widetilde{F}_5$ must be imposed as a constraint in addition to the equations of motion that follow from (5). The $\widetilde{F}_5$ Bianchi identity reads:

其中 κ_{10} 是十维引力耦合，爱因斯坦框架度规 g_{MN} 与弦框架度规 $\hat{g}_{MN}$ 满足关系 $g_{MN} = \sqrt{\mathrm{Im}\,\tau}\hat{g}_{MN}$ 。除了由 (5) 导出的运动方程外，还必须将自对偶条件 $\widetilde{F}_5 = \star_{10}\widetilde{F}_5$ 作为约束补充进来。$\widetilde{F}_5$ 比安基恒等式为:

$$\mathrm{d}\widetilde{F}_5 = H_3 \wedge F_3 + \rho_{\mathrm{loc}}^{\mathrm{D3}} \tag{6}$$

D-branes carry Ramond-Ramond charge, and in particular C_4 couples to the worldvolume of a D3-brane, which is why the source term is denoted by $\rho_{\mathrm{loc}}^{\mathrm{D3}}$: it captures the D3-brane charge $Q_{\mathrm{loc}}^{\mathrm{D3}} \ := \ \int\rho_{\mathrm{loc}}^{\mathrm{D3}}$ from localized sources, including, but not limited to, actual D3-branes [6] .

D 膜携带拉蒙-拉蒙荷，具体来说 C_4 与 D3 膜的世界体积耦合，因此源项记为 $\rho_{\mathrm{loc}}^{\mathrm{D3}}$: 它捕获了来自定域源的 D3 膜荷 $Q_{\mathrm{loc}}^{\mathrm{D3}} \ := \ \int\rho_{\mathrm{loc}}^{\mathrm{D3}}$ ，这些定域源包括但不限于实际的 D3 膜 [6] 。

Scalings and Perturbative Expansions

标度与微扰展开

The action (5) receives corrections in two different expansions: the α' and string loop expansions. These two perturbative expansions can be traced to the two scaling symmetries of the tree-level bulk action (5) [11,12]. We consider the transformations:

作用量 (5) 会在两种不同展开下得到修正: α' 展开和弦圈展开。这两种微扰展开可以追溯到树级体作用量 (5) 的两种标度对称性 [11,12]。我们考虑如下变换:

(i) $\tau \to a^2\tau,\ G_3 \to aG_3,\ S_{10} \to S_{10}$,
(ii) $g_{MN} \to \lambda^{\nu} g_{MN},\ \tau \to \lambda^{-2\nu}\tau,\ \widetilde{F}_5 \to \lambda^{2\nu}\widetilde{F}_5,\ S_{10} \to \lambda^{4\nu} S_{10}$.

(ii) $g_{MN} \to \lambda^{\nu} g_{MN},\ \tau \to \lambda^{-2\nu}\tau,\ \widetilde{F}_5 \to \lambda^{2\nu}\widetilde{F}_5,\ S_{10} \to \lambda^{4\nu} S_{10}$ 。

The transformation (i) is a symmetry contained in the general $SL(2,\mathbb{R})$ invariance of (5), which acts as

变换 (i) 是 (5) 的广义 $SL(2,\mathbb{R})$ 不变性中包含的一种对称性，其作用形式为

$$\tau \to \frac{a\tau + b}{c\tau + d},\ G_3 \to \frac{G_3}{c\tau + d}. \tag{7}$$

On the other hand, although (ii) leaves invariant the tree-level equations of motion, it is not a symmetry of the action: S_{10} is not invariant under (ii), but scales.

另一方面，尽管 (ii) 保持树级运动方程不变，它并非作用量的对称性: S_{10} 在 (ii) 变换下不保持不变，而是会发生标度变化。

The transformations (i) and (ii) can be combined, writing $a = \lambda^{\omega}$, as

我们可以将变换 (i) 和 (ii) 结合，写作 $a = \lambda^{\omega}$ ，形式如下

$$g_{MN} \to \lambda^{\upsilon} g_{MN},\ \tau \to \lambda^{2(\omega-\upsilon)}\tau,\ G_3 \to \lambda^{\omega} G_3,\ F_5 \to \lambda^{2\upsilon} F_5,$$

$$S_{10} \to \lambda^{4\upsilon} S_{10} \tag{8}$$

Both scaling symmetries are broken by quantum and α' corrections, and as we will now explain, the corresponding perturbative expansions can be organized by the amount that the scaling symmetries are broken.

两种标度对称性都会被量子修正和 α' 修正破坏，我们接下来会说明，相应的微扰展开可以按照标度对称性的破坏程度来组织。

The string loop expansion is in powers of $g_s = \langle e^{\phi} \rangle$, and the α' expansion is essentially a derivative expansion. We write the general action, including all perturbative corrections, schematically as

弦圈展开是按 $g_s = \langle e^{\phi} \rangle$ 的幂次展开，而 α' 展开本质上是导数展开。我们将包含所有微扰修正的一般作用量示意性地写作

$$S_{10} = \sum_{m,n=0}^{\infty} (\alpha')^m g_s^n S_{10}^{(m,n)} \tag{9}$$

with a similar expansion for S_{loc} . Here $S_{10}^{(m,n)}$ can be written as [11]

S_{loc} 也有类似的展开。这里 $S_{10}^{(m,n)}$ 可以写作 [11]

$$S_{10}^{(m,n)} \equiv S_{10}^{(p,r,n)} \propto \int \sqrt{-g} \left(\frac{1}{\operatorname{Im} \tau} \right)^{(2n-p+r+1)/2}$$

$$(g^{\circ\circ} \mathcal{R}^{\circ}_{\circ\circ\circ})^p [g^{\circ\circ} g^{\circ\circ} g^{\circ\circ} G_{\circ\circ\circ} G_{\circ\circ\circ}]^r + \dots \tag{10}$$

---

[6] Our conventions are such that D3-branes and anti-D3-branes carry positive) and negative D3-brane charge $Q_{\text{loc}}^{\text{D3}}$, respectively.

[6] 我们的约定是:D3 膜和反 D3 膜分别携带正 D3 膜电荷 $Q_{\text{loc}}^{\text{D3}}$ 和负 D3 膜电荷 $Q_{\text{loc}}^{\text{D3}}$ 。

---

where $\circ$ determines the appropriate index structure, and we have split $m = p + r - 1$ since it is convenient to separate the m dependence in terms of powers of the curvature $\mathcal{R}$ and powers of G_3 . We have not written explicitly the terms depending on F_5 or on higher derivatives of the dilaton, which we include in the ellipses. The p and r powers in $\operatorname{Im} \tau$ appear because we are writing the action in Einstein frame.

其中 $\circ$ 确定了相应的指标结构，我们拆分了 $m = p + r - 1$ ，这是因为按曲率 $\mathcal{R}$ 的幂次和 G_3 的幂次分离出 m 依赖会更方便。我们没有显式写出依赖于 F_5 或涨落子更高阶导数的项，将它们归入省略号中。$\operatorname{Im} \tau$ 中出现 p 和 r 幂次是因为我们在爱因斯坦框架下写出该作用量。

Notice that both $p = 1, r = 0$ and $p = 0, r = 1$ give tree-level results for $m = 0, n = 0$. Also, taking $m = 3$ with all combinations of $p, r = 0, 1, 2, 3$ satisfying $p + r = 4$ reproduces the known α'^3 corrections (starting with $\mathcal{R}^4$, etc.) that are known to be the first nonvanishing α' corrections (see for instance [13-15]). Terms with smaller values of m cancel due to supersymmetry. The corrected actions $S_{10}^{(m,n)}$ transform under the scalings as

注意 $p = 1, r = 0$ 和 $p = 0, r = 1$ 都能给出 $m = 0, n = 0$ 的树级结果。此外，取 $m = 3$ 并令 $p, r = 0, 1, 2, 3$ 的所有组合满足 $p + r = 4$ ，即可得到已知的 α'^3 修正 (从 $\mathcal{R}^4$ 等项开始)，这些都是首个非零的 α' 修正 (例如参见 [13-15])。更小的 m 值对应的项会因超对称对消。修正后的作用量 $S_{10}^{(m,n)}$ 在标度变换下的变换形式为

$$S_{10}^{(m,n)} \to \lambda^{4v-2n(w-v)+m(w-2v)} S_{10}^{(m,n)}. \tag{11}$$

We can then use the scaling transformations as a bookkeeping tool to identify the different corrections $S_{10}^{(m,n)}$. This will be useful in section ”Perturbative Corrections” in order to uncover the structure of the four-dimensional effective actions in string compactifications.

我们可以将标度变换作为记账工具来标识不同的修正 $S_{10}^{(m,n)}$ 。这在“微扰修正”一节中会很有用，有助于我们揭示弦紧化下四维有效作用量的结构。

Calabi-Yau Compactifications

卡拉比-丘紧致化

In compactifying type IIB string theory on a Calabi-Yau threefold X , one needs to specify the three-form fluxes H_3 and F_3 [16, 17] . Dirac quantization requires that (in units where $(2\pi)^2\alpha' \equiv \ell_s^2 = 1$)

将 IIB 型弦理论紧致化在卡拉比-丘三 fold X 上时，需要指定三形式 flux H_3 和 F_3 [16, 17] 。狄拉克量子化要求 (在 $(2\pi)^2\alpha' \equiv \ell_s^2 = 1$ 为单位的约定下)

$$F_3, H_3 \in H^3(X, \mathbb{Z}) \,, \tag{12}$$

and so the data of a choice of fluxes is a set of integers.

因此 flux 选择的自由度对应一组整数。

Gauss’s law presents an immediate obstacle to constructing flux compactifi-cations. Many sources of stress energy, including D-branes as well as certain flux configurations, carry positive charge under Ramond-Ramond p -form gauge symmetries. In a compact space, the total charge must be zero, so in the absence of sources with negative charge, a consistent compactification must have no charged sources whatsoever. Specifically, integrating (6), we find the constraint:

高斯定律给构建 flux 紧致化带来了一个直接的障碍。包括 D 膜在内的许多应力能量源，以及特定的 flux 构型，都在拉蒙德-拉蒙德 p 形式规范对称性下携带正电荷。紧致空间中的总电荷必须为零，因此如果不存在带负电的源，自洽的紧致化根本就不能包含任何带电源。具体来说，对 (6) 积分后我们得到约束:

$$0 = Q_{\rm loc}^{\rm D3} + \int H_3 \wedge F_3 = Q_{\rm loc}^{\rm D3} + Q_{\rm flux}^{\rm D3}. \tag{13}$$

Thus, a solution in which $Q_{\rm flux}^{\rm D3}$ is positive - and we will see soon that these are precisely the solutions of interest for moduli stabilization - is then possible only in the presence of localized sources of negative D3-brane charge.

因此，只有存在带负 D3 膜电荷的局域源时，才可能存在 $Q_{\rm flux}^{\rm D3}$ 为正的解——我们很快就会看到，这类解正是模稳定中我们感兴趣的解。

A way forward is provided by orientifolds. Orientifold planes are non-dynamical objects that have negative tension, and carry negative charge with respect to Ramond-Ramond potentials. As an example, consider type IIB string theory compactified on a Calabi-Yau threefold X_6 . At certain loci in the complex structure moduli space of X_6 , the manifold may admit a holomorphic involution:

> 定向模为我们提供了一条可行的思路。定向平面是非动力学对象，具有负张力，在拉蒙德-拉蒙德势下携带负电荷。举个例子，考虑紧致化在卡拉比-丘三 fold X_6 上的 IIB 型弦理论。在 X_6 的复结构模空间的特定轨迹上，流形可以存在全纯对合:

$$\sigma : X_6 \to X_6 \tag{14}$$

whose fixed loci are points and/or divisors in X_6 . If the action of σ is paired with orientation reversal Ω_{WS} on the string worldsheet and multiplication by $(-1)^{F_L}$, with F_L the left-moving fermion number,

> 其不动轨迹是 X_6 中的点和/或除子。如果 σ 的作用，与弦世界面上的取向反转 Ω_{WS} 以及对 $(-1)^{F_L}$ 的乘法配对，其中 F_L 是左行费米子数,

$$\mathcal{O} := \sigma\Omega_{\mathrm{WS}}(-1)^{F_L} \tag{15}$$

then the configuration that results from projecting onto states with $\mathcal{O} = +1$ is called an O3/O7 orientifold. The fixed loci, which are known as O3-planes and O7-planes, carry negative [7] charge with respect to C_4 , i.e., they contribute negatively to $Q^{\mathrm{D3}}_{\mathrm{loc}}$.

> 那么投影到具有 $\mathcal{O} = +1$ 态后得到的构型就称为 O3/O7 定向模。不动轨迹被称为 O3 平面和 O7 平面，在 C_4 下携带负 [7] 电荷，也就是说它们对 $Q^{\mathrm{D3}}_{\mathrm{loc}}$ 的贡献为负。

The orientifold projection removes half of the supercharges preserved by X_6 , leading to a theory with $\mathcal{N} = 1$ supersymmetry in four dimensions. From each hypermultiplet arising in type IIB compactification on X_6 , a chiral multiplet survives the projection, while from each $\mathcal{N} = 2$ vector multiplet, either an $\mathcal{N} = 1$ vector multiplet or a chiral multiplet survives. The action of $\mathcal{O}$ on Dolbeault cohomology classes defines even and odd eigenspaces $H^{p,q}_{\pm}$ with corresponding dimensions $h^{p,q}_{\pm}$. The $h^{1,1}$ hypermultiplets yield $h^{1,1}_{+}$ Kähler moduli T_a and $h^{1,1}_{-}$ two-forms G_α , while the $h^{2,1}$ hypermultiplets yield $h^{2,1}_{-}$ complex structure moduli z_i and $h^{2,1}_{+}$ vector multiplets V_r : see Table 1. For simplicity of presentation, we will mostly discuss orientifolds with $h^{1,1}_{-} = h^{2,1}_{+} = 0$, in which the massless closed-string scalar fields are the geometric moduli T_a and z_i , and the axiodilaton τ[8] .

> 定向模投影会移除一半被 X_6 保留的超荷，最终得到的四维理论具有 $\mathcal{N} = 1$ 超对称。IIB 在 X_6 上紧致化得到的每个超多重态中，有一个手征多重态在投影后保留下来，而每个 $\mathcal{N} = 2$ 矢量多重态中，则会保留一个 $\mathcal{N} = 1$ 矢量多重态或一个手征多重态。$\mathcal{O}$ 对 Dolbeault 上同调类的作用定义了偶本征空间和奇本征空间 $H^{p,q}_{\pm}$ ，对应的维数为 $h^{p,q}_{\pm}$ 。$h^{1,1}$ 个超多重态给出 $h^{1,1}_{+}$ 个凯勒模 T_a 和 $h^{1,1}_{-}$ 个二形式 G_α ，而 $h^{2,1}$ 个超多重态给出 $h^{2,1}_{-}$ 个复结构模 z_i 和 $h^{2,1}_{+}$ 个矢量多重态 V_r : 参见表 1。为了简化表述，我们大多会讨论满足 $h^{1,1}_{-} = h^{2,1}_{+} = 0$ 的定向模，这类定向模中无质量闭弦标量场是几何模 T_a 和 z_i ，以及轴子 dilaton τ[8] 。

Table 1$\mathcal{N} = 1$ multiplets in an O3/O7 orientifold

表 1$\mathcal{N} = 1$ O3/O7 定向模中的多重态

Field	Symbol	Range
Kähler moduli	T_a	$a = 1, \ldots, h_+^{1,1}$
Complex structure moduli	z_i	$i = 1, \ldots, h_-^{2,1}$
Two-forms	G_α	$\alpha = 1, \ldots, h_-^{1,1}$
Vector multiplets	V_r	$r = 1, \ldots, h_+^{2,1}$
Axiodilaton	τ	-

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[7] In certain cases, O7-planes that break supersymmetry can carry positive charge: see, e.g.,[18]. [8] See, e.g.,[18-24] for discussions of the geometry and physics of orientifolds with $h_-^{1,1} \neq 0$.

[7] 在某些情况下，破坏超对称的 O7-平面可以带正电荷: 例如参见文献 [18]。[8] 有关带 $h_-^{1,1} \neq 0$ 的定向模的几何与物理讨论，参见例如文献 [18-24]。

Compactification of type IIB string theory on an O3/O7 orientifold [9] of a Calabi-Yau threefold is a promising starting point for finding flux compactifications that involve nontrivial sources and that preserve at most $\mathcal{N} = 1$ supersymmetry. From there, one can further aim to find non-supersymmetric solutions without moduli.

IIB 型弦理论在 O3/O7 定向模 [9] (对应一个卡拉比-丘三维流形) 上紧化，是寻找包含非平凡源且最多保留 $\mathcal{N} = 1$ 超对称的流量紧化的一个很有前景的出发点。在此基础上，我们还可以进一步寻找无模的非超对称解。

Four-Dimensional $\mathcal{N} = 1$ Supersymmetric Action

四维 $\mathcal{N} = 1$ 超对称作用量

We can now see how the four-dimensional action transforms under the scaling transformations and how it can be expanded in powers of α' and g_s . For this we need to use that the volume of X_6 scales as

我们现在可以分析四维作用量在标度变换下如何变换，以及它如何按 α' 和 g_s 的幂次展开。为此我们需要用到 X_6 的体积满足如下标度关系

$$\mathcal{V} = \int_{X_6} \mathrm{d}^6 x \sqrt{g^{(6)}} \to \lambda^{3v} \mathcal{V}, \tag{16}$$

where the volume is measured in units of the string length $\ell_s = 2\pi\sqrt{\alpha'}$. Then, the four-dimensional Einstein-frame metric $g_{\mu\nu}^E$, Kähler moduli τ_a , complex structure moduli z_i , and Lagrangian $\mathcal{L}$ scale as

其中体积以弦长 $\ell_s = 2\pi\sqrt{\alpha'}$ 为单位度量。此时，四维爱因斯坦框架度规 $g_{\mu\nu}^E$ 、凯勒模 τ_a 、复结构模 z_i 和拉氏量 $\mathcal{L}$ 的标度关系为

$$g_{\mu\nu}^{E}=\mathcal{V}g_{\mu\nu}\rightarrow\lambda^{4\upsilon}g_{\mu\nu}^{E},\ \tau_a\rightarrow\lambda^{2\upsilon}\tau_a,\ z_i\rightarrow z_i,\ \mathcal{L}\rightarrow\lambda^{4\upsilon}\mathcal{L}.$$

(17)

In this article we are interested in $\mathcal{N}=1$ supersymmetric compactifications. The general couplings of four-dimensional supergravity to matter fields were computed in [25, 26] . Up to two derivatives and neglecting the contributions of vector multiplets, the action for an arbitrary number of chiral superfields Φ_M coupled to supergravity is specified by a superpotential $W(\Phi_M)$ and a Kähler potential $K\left(\Phi_M,\overline{\Phi}_{\bar{N}}\right)$, with W a holomorphic function and K a real analytic function of the chiral superfields Φ_M .

本文我们关注 $\mathcal{N}=1$ 维超对称紧化。四维超引力与物质场的一般耦合已在 [25, 26] 中计算得到。在不超过二阶导数、忽略矢量多重态贡献的前提下，任意多个手征超场 Φ_M 耦合到超引力的作用量由超势 $W(\Phi_M)$ 和凯勒势 $K\left(\Phi_M,\overline{\Phi}_{\bar{N}}\right)$ 确定，其中 W 是手征超场 Φ_M 的全纯函数，K 是手征超场 Φ_M 的实解析函数。

We write the four-dimensional Lagrangian in the superconformal formalism [25, 26] in terms of the superpotential W and Kähler potential K with conformal compensator $\mathcal{C}$ as

我们在超共形形式体系 [25, 26] 中，利用共形补偿场 $\mathcal{C}$ ，将四维拉氏量通过超势 W 和凯勒势 K 写为

$$\frac{\mathcal{L}}{\sqrt{-g^E}}=\int \mathrm{d}^4\theta\overline{\mathcal{C}}\mathcal{C}e^{-K/3}+\left(\int \mathrm{d}^2\theta\mathcal{C}^3W+\text{ h.c.}\right). \tag{18}$$

Using the scaling of the ten-dimensional actions given in (11), we infer that the perturbative corrections to K can be written as [11]

利用 (11) 中给出的十维作用量的标度关系，我们推导出 K 的微扰修正可以写为 [11]

$$\left(e^{-K/3}\right)^{(m,n)}\rightarrow\lambda^{\frac{2}{3}(\omega+2\upsilon)-2(\omega-\upsilon)n+(\omega-2\upsilon)m}\left(e^{-K/3}\right)^{(m,n)}, \tag{19}$$

---

[9] We will discuss other string theories in section "Beyond IIB". Within type IIB string theory, an advantage of O3/O7 orientifolds compared to O5/O9 orientifolds is the possibility of conformally Calabi-Yau flux compactifications, as we will explain in section "ISD Flux Compactifications".

[9] 我们将在“IIB 理论之外”一节讨论其他弦理论。在 IIB 型弦理论框架内，O3/O7 定向轨相比 O5/O9 定向轨的一个优势是可以存在共形卡丘-姚流形通量紧化，我们会在“ISD 通量紧化”一节解释这一点。

Here we have used that the weight of θ is υ in order for the fermionic kinetic terms to transform appropriately, and thus the compensator $\mathcal{C}$ carries weight $-(\omega+2\upsilon)/3$.

此处我们利用了 θ 的权重为 υ ，这是费米子动能项正确变换所要求的，因此补偿场 $\mathcal{C}$ 的权重为 $-(\omega+2\upsilon)/3$ 。

The superpotential W (which scales as $W \to \lambda^{\omega} W$) is holomorphic and therefore receives no perturbative corrections whatsoever [10] , whereas the Kähler potential K is not holomorphic and is subject to both perturbative corrections - as can be seen from (19) - and non-perturbative corrections. Thus, we can write:

超势 W (其标度为 $W \to \lambda^{\omega} W$) 是全纯的，因此完全不存在微扰修正 [10]；而凯勒势 K 不是全纯的，会同时受到微扰修正 (如 (19 式所示) 和非微扰修正。因此我们可以写出:

$$W = W_{\text{tree}} + W_{\text{np}},\ K = K_{\text{tree}} + K_{\text{pert}} + K_{\text{np}}. \tag{20}$$

After introducing the central ansatz for type IIB flux compactifications in section "ISD Flux Compactifications", we will examine each of W_{tree} (section "Flux Superpotential"), W_{np} (section "Non-perturbative Superpotential"), K_{tree} (section "Tree Level"), and K_{pert} (section "Perturbative Corrections"). We will not treat K_{np} , which is poorly understood.

在“ISD 通量紧化”一节引入 IIB 型通量紧化的中心假设后，我们将逐一考察 W_{tree}（“通量超势”一节)、W_{np}（“非微扰超势”一节)、K_{tree}（“树图水平”一节) 和 K_{pert}（“微扰修正”一节)。我们不会讨论 K_{np}，该部分目前还缺乏清晰认知。

Knowing K and W , we can compute all the relevant couplings for the bosonic and fermionic components of the chiral superfields Φ_M . In particular, the F-term scalar potential, which is the key quantity to compute in order to find the vacuum states, depends on W and K as follows:

知道 K 和 W 后，我们就可以计算手征超场 Φ_M 的所有玻色子和费米子分量的相关耦合。特别是，寻找真空态需要计算的核心量——F 项标量势，它对 W 和 K 的依赖关系如下:

$$V_F = e^K \left[K^{M\bar{N}} D_M W \bar{D}_{\bar{N}} \bar{W} - 3|W|^2 \right], \tag{21}$$

where $K^{M\bar{N}}$ is the inverse of the Kähler metric $K_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$, and $D_M W = \partial_M W + K_M W$ is the Kähler-covariant derivative, with $K_M = \partial_M K$. Here ∂_M refers to the derivative with respect to the scalar component ϕ_M of the superfield Φ_M , and we have adopted units in which $M_{\text{pl}} = 1$.

其中 $K^{M\bar{N}}$ 是凯勒度量 $K_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$ 的逆,$D_M W = \partial_M W + K_M W$ 是凯勒协变导数,满足 $K_M = \partial_M K$ 。此处 ∂_M 指对超场 Φ_M 的标量分量 ϕ_M 求导，我们采用了单位制，其中 $M_{\text{pl}} = 1$ 。

ISD Flux Compactifications

ISD 通量紧致化

In a flux compactification [11] , the metric of the internal space is not Ricci-flat, because the stress-energy of fluxes drives deviations from the Calabi-Yau vacuum configuration. However, in an important class of type IIB flux compactifications, the metric is conformal to a Calabi-Yau metric, differing only by a warp factor. To see this, we consider the warped ansatz:

在通量紧致化 [11] 中, 内部空间的度量不是里奇平坦的, 因为通量的能动张量会导致其偏离卡拉比-丘真空构型。但在一类重要的 IIB 型通量紧致化中，该度量共形于卡拉比-丘度量，仅相差一个翘曲因子。为说明这一点，我们考虑如下翘生育定猜想:

$$ds^2 = e^{2A(y)} g_{\mu\upsilon}(x)\, \mathrm{d}x^\mu \mathrm{d}x^\upsilon + e^{-2A(y)} g_{mn}(y)\, \mathrm{d}y^m\ \mathrm{d}y^n, \tag{22}$$

with g_{mn} a Riemannian metric on a compact space X_6 that admits a Calabi-Yau metric g_{mn}^{CY} . In a Calabi-Yau vacuum solution, $A(y)$ is trivial and $g_{mn} = g_{mn}^{\mathrm{CY}}$ on X_6 , whereas in the presence of general sources, g_{mn} is unrelated to g_{mn}^{CY} . Defining the Hodge star $\star$ constructed from the metric g_{mn} , one finds that $\star^2 = -1$, so $\star$ has eigenvalues $\pm i$. Writing the three-form fluxes F_3 and H_3 in the complex combination

其中 g_{mn} 是紧致空间 X_6 上的黎曼度量，该空间容许卡拉比-丘度量 g_{mn}^{CY} 。在卡拉比-丘真空解中, $A(y)$ 是平凡的且满足 $g_{mn} = g_{mn}^{\mathrm{CY}}$ 于 X_6 上；而存在一般源时，g_{mn} 与 g_{mn}^{CY} 无关。定义由度量 g_{mn} 构造的霍奇星算子 $\star$ ，可推得 $\star^2 = -1$ ，因此 $\star$ 的本征值为 $\pm i$ 。将三形式流 F_3 与 H_3 写为复组合

$$G_3 \ :=\ F_3 - \tau H_3, \tag{23}$$

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[10] The original argument uses the shift symmetry for the dilaton field [27-29], but for flux compactifications the dilaton appears in the superpotential, and so a more refined proof of the non-renormalization theorem, given in [30], is needed.

[10] 原始论证用到了 dilaton 场的平移对称性 [27-29]，但在通量紧致化中，dilaton 会出现在超势里，因此需要采用文献 [30] 给出的更精细的非重整化定理证明。

[11] In this review we concentrate on fluxes of antisymmetric tensor fields. More general fluxes, including geometric fluxes (see, e.g., [31]) and non-geometric fluxes [32, 33] are possible: see the review [34].

[11] 本综述聚焦于反对称张量场的通量。更一般的通量，包括几何通量 (例如参见文献 [31]) 和非几何通量 [32, 33] 也是存在的: 参见综述文献 [34]。

---

with τ the axiodilaton, we can decompose G_3 into $+i$ and $-i$ eigenspaces of $\star$:

其中 τ 为轴子膨胀子，我们可将 G_3 分解为 $\star$ 的 $+i$ 和 $-i$ 本征空间:

$$G_\pm \ :=\ G_3 \mp i \star G_3 \tag{24}$$

which are termed imaginary self-dual (ISD) and imaginary anti-self-dual (IASD), respectively.

它们分别被称为虚自对偶 (ISD) 和虚反自对偶 (IASD)。

Consider type IIB string theory compactified on an O3/O7 orientifold of a Calabi-Yau threefold, and containing only ISD fluxes, D3-branes, D7-branes, O3-planes, and O7-planes, without IASD fluxes and without antibranes. Such a configuration is called an ISD compactification.

考虑 IIB 型弦论在卡拉比-丘三维流形的 O3/O7 定向轨形上紧化，其中仅包含 ISD 通量、D3 膜、D7 膜、O3 平面和 O7 平面，不含 IASD 通量与反膜。这类构型被称为 ISD 紧化。

Several key properties of ISD compactifications were recognized by Giddings, Kachru, and Polchinski [17]. First, the Einstein equations for (22) are solved by $g_{mn} = g_{mn}^{\mathrm{CY}}$ with a generally nontrivial [12] warp factor $A(y)$. That is, the metric $e^{-2A(y)} g_{mn}(y)$ on the internal space is conformally Calabi-Yau. Second, the classical solution at leading order in the α' expansion enjoys a dilatation symmetry: the size of X_6 is a modulus (see section "Tree Level"). Third, generic ISD fluxes give masses to the complex structure moduli of X_6 , and to the axiodilaton. This is easy to see from the ten-dimensional action:

吉丁斯 (Giddings)、卡楚 (Kachru) 和波利钦斯基 (Polchinski)[17] 已经发现了 ISD 紧致化的若干关键性质。首先，(22) 式的爱因斯坦方程组可由 $g_{mn} = g_{mn}^{\mathrm{CY}}$ 求解，得到通常非平凡的 [12] 翘曲因子 $A(y)$ 。也就是说，内空间上的度规 $e^{-2A(y)} g_{mn}(y)$ 共形于卡拉比-丘空间。其次，领头阶经典解在 α' 展开下具有伸缩对称性: X_6 的尺寸是一个模 (参见“树水平”章节)。第三，一般的 ISD 通量会为 X_6 的复结构模以及轴 dilaton 赋予质量。这一点从十维作用量中很容易看出:

$$S_{10} \supset \int_{X_6} G_3 \wedge \star \overline{G_3} \tag{25}$$

in which the Hodge star, which depends on the metric g_{mn} , couples to the fluxes [13] .

其中依赖于度量 g_{mn} 的霍奇星与流 [13] 耦合。

Dimensional reduction of an ISD compactification leads to an $\mathcal{N} = 1$ super-symmetric effective action in four dimensions. On general grounds the resulting superpotential W and Kähler potential K depend on the moduli as

ISD 紧致化的维数约化会在四维下得到一个 $\mathcal{N} = 1$ 超对称有效作用量。一般而言，得到的超势 W 和凯勒势 K 对模的依赖关系为

$$W = W(z_i, T_a, \tau), \; K = K\left(z_i, \bar{z}_i, T_a, \bar{T}_a, \tau, \bar{\tau}\right), \tag{26}$$

However, as we will now see, there is a great deal of structure in W and K that can be exploited in searching for vacua.

然而，正如我们接下来将看到的，在 W 和 K 中存在大量可用于搜索真空的结构。

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[12] The dynamics of the warp factor is analyzed in [35, 36] .

[12] 翘曲因子的动力学在 [35, 36] 中得到分析。

[13] One might wonder how to choose quantized fluxes that are sufficiently generic to stabilize all, rather than just some, of the complex structure moduli. We will address this below.

[13] 人们可能会疑惑，该如何选择足够一般的量子化通量，以稳定全部而非仅部分复结构模。我们将在下文讨论这个问题。

Superpotential

超势

Flux Superpotential

流超势

The classical Gukov-Vafa-Witten flux superpotential is [37]

经典古科夫-瓦法-威滕流超势为 [37]

$$W_{\text{tree}} \equiv W_{\text{flux}} := \sqrt{\frac{2}{\pi}} \int_{X_6} G_3 \wedge \Omega, \tag{27}$$

where Ω is the holomorphic $(3,0)$ form on X_6 , and we have adopted a convenient normalization.

其中 Ω 是 X_6 上的全纯 $(3,0)$ 形式，我们采用了便捷归一化方式。

To better understand this expression, we take α_A, β^A to be a symplectic basis of $H^3(X_6, \mathbb{Z})$, with $A = 0, \ldots, h^{2,1}$ and with $\int_{X_6} \alpha_A \wedge \beta^B = \delta_A^B$, and we introduce the periods:

为更好理解该表达式，我们取 α_A, β^A 作为 $H^3(X_6, \mathbb{Z})$ 的辛基，满足 $A = 0, \ldots, h^{2,1}$ 和 $\int_{X_6} \alpha_A \wedge \beta^B = \delta_A^B$ ，随后引入周期:

$$\vec{\Pi} := \begin{pmatrix} \int \Omega \wedge \beta_A \\ \int \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \mathcal{F}_A \\ z^A \end{pmatrix}. \tag{28}$$

The periods z^A furnish local projective coordinates on complex structure moduli space: in a patch where $z^0 \neq 0$, we can fix the normalization of Ω to set $z^0 = 1$, and then use $\{z_i\}, i = 1, \ldots, h^{2,1}$ as independent coordinates.

周期 z^A 提供了复结构模空间上的局部射影坐标: 在 $z^0 \neq 0$ 所在的坐标 patch 中，我们可以固定 Ω 的归一化以设定 $z^0 = 1$ ，再将 $\{z_i\}, i = 1, \ldots, h^{2,1}$ 用作独立坐标。

We write the integrals of the three-form fluxes as

我们将三形式流的积分记为

$$\vec{f} := \left(\int F_3 \wedge \alpha_A, \int F_3 \wedge \alpha^A\right),\ \vec{h} := \left(\int H_3 \wedge \alpha_A, \int H_3 \wedge \alpha^A\right). \tag{29}$$

By Dirac quantization, $\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$. Introducing the symplectic matrix

根据狄拉克量子化条件，$\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$ 。引入辛矩阵

$$\sum := \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}, \tag{30}$$

we can express (27) as the pairing

我们可以将式 (27) 表示为内积

$$W_{\text{flux}}(\tau, z_i) = \sqrt{\frac{2}{\pi}} \vec{\Pi}^{\mathsf{T}} \cdot \sum \cdot \left(\vec{f} - \tau\vec{h}\right). \tag{31}$$

Thus, the flux superpotential is determined by the periods, the integer flux quanta $\vec{f}$ and $\vec{h}$, and the axiodilaton τ .

因此，流超势由周期、整数量子流 $\vec{f}$ 和 $\vec{h}$ ，以及轴子- dilaton τ 共同确定。

Non-perturbative Superpotential

非微扰超势

The Kähler moduli T_a do not appear in the classical flux superpotential W_{flux} , and indeed do not appear in W at any perturbative order in α' or g_s . We explained this fact in general terms in section "Scalings and Perturbative Expansions", but we will now be more specific about the dependence on T_a .

凯勒模 T_a 不会出现在经典通量超势 W_{flux} 中，实际上在 α' 或 g_s 的任意微扰阶下都不会出现在 W 中。我们在“标度与微扰展开”章节已经对该事实做了一般性说明，现在我们将具体讨论对 T_a 的依赖关系。

Suppose that $\{D_a\}, a = 1, \ldots, h_+^{1,1}$ is a basis of $H_4(X_6, \mathbb{Z})$ consisting of effective divisors (holomorphic four-cycles), with complexified volumes T_a given by

假设 $\{D_a\}, a = 1, \ldots, h_+^{1,1}$ 是由有效除子 (全纯四维环面) 构成的 $H_4(X_6, \mathbb{Z})$ 的一组基，其复化体积 T_a 由下式给出:

$$T_a := \frac{1}{2}\int_{D_a} J \wedge J + i\int_{D_a} C_4 \equiv \tau_a + i\theta_a, \tag{32}$$

where J is the Kähler form, and we have introduced the axion θ_a and the volume modulus (or saxion) τ_a .

其中 J 是凯勒形式，我们已经引入了轴子 θ_a 和体积模 (也叫 saxion) τ_a 。

The action (5) is invariant under continuous shifts:

作用量 (5) 在连续平移下不变:

$$\theta_a \to \theta_a + c, \; c \in \mathbb{R}. \tag{33}$$

This shift symmetry descends from the gauge redundancy of the C_4 kinetic term proportional to $|\widetilde{F}_5|^2$, and is known as a Peccei-Quinn (PQ) symmetry [14] . Because the string worldsheet carries no Ramond-Ramond charge, the PQ symmetry (33) of C_4 is not broken at any order in string perturbation theory, or at any order in α' .

这种平移对称性源自正比于 $|\widetilde{F}_5|^2$ 的 C_4 动能项的规范冗余，被称为佩西-奎因 (PQ) 对称性 [14] 。由于弦世界面不携带拉蒙德-拉蒙德荷，C_4 的 PQ 对称性 (33) 在弦微扰论的任意阶、乃至 α' 的任意阶都不会破缺。

Any correction to the effective action at some perturbative order in the α' expansion would have to scale as a negative power of (some of) the τ_a , in order to vanish in the limit of infinite volume. But on general grounds, the superpotential must be holomorphic, and so it can only depend on cycle volumes via the complex combination $T_a = \tau_a + i\theta_a$: the T_a are the "good Kähler coordinates" on Kähler moduli space. But no term polynomial in $1/T_a$ is invariant under the PQ symmetry (33), which remains unbroken to all perturbative orders, and so the superpotential must have no perturbative dependence on the Kähler moduli T_a .

α' 展开中任意微扰阶对有效作用量的修正都必须标度为 τ_a (中部分成员) 的负次幂，才能在无穷体积极限下消失。但一般来说，超势必须是全纯的，因此它只能通过复组合 $T_a = \tau_a + i\theta_a$ 依赖于环面体积: T_a 才是凯勒模空间上的“好凯勒坐标”。但在保持 PQ 对称性 (33) 不被破坏 (该对称性在所有微扰阶都保持不破缺) 的前提下，不存在 $1/T_a$ 的多项式项满足不变性，因此超势对凯勒模 T_a 没有微扰依赖。

The symmetry (33) can be broken by non-perturbative effects from Euclidean D3-branes, causing W to depend on exponentials of the T_a , as we now explain. We suppose that $D = c^a D_a$, with $c^a \in \mathbb{Z}$, is a four-cycle in X_6 , but is not necessarily an effective divisor. Then, the semiclassical action of a Euclidean D3-brane wrapping D is

该对称性 (33) 可以被欧氏 D3 膜带来的非微扰效应破缺，使得 W 依赖于 T_a 的指数形式，我们接下来解释这一点。我们假设 $D = c^a D_a$ (满足 $c^a \in \mathbb{Z}$) 是 X_6 中的一个四维环面，但不一定是有效除子。那么，包裹 D 的欧氏 D3 膜的半经典作用量为:

$$\frac{1}{2\pi} S_{\mathrm{ED3}} = \mathrm{Vol}\,(D) + i \int_D C_4. \tag{34}$$

If D is an effective divisor, it is calibrated by the Kähler form J , and so obeys

如果 D 是有效除子，它会被凯勒形式 J 校准，因此满足:

$$\mathrm{Vol}(D) = \frac{1}{2}\int_D J \wedge J \tag{35}$$

so that

因此

$$\frac{1}{2\pi} S_{\mathrm{ED3}} = c^a T_a \tag{36}$$

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14 The Peccei-Quinn symmetry is discussed from a slightly different perspective in section ”Axions in String Theory”.

注 14: 佩西-奎因对称性在“弦论中的轴子”章节从略有不同的角度做了讨论。

Euclidean D3-branes wrapping such a divisor contribute to the superpotential, rather than to the Kähler potential or higher F-terms, if and only if the associated Dirac operator on D has exactly two zero modes. These zero modes can be counted in terms of the dimensions of the orientifold-graded sheaf cohomology groups $H^i_\pm(D, \mathcal{O}_D)$, with $i = 0, 1, 2$. If D is smooth, effective, and obeys the rigidity condition [15]

当且仅当 D 上的关联狄拉克算子恰好有两个零模时，包裹这类除子的欧氏 D3 膜才会对超势产生贡献 (而非对凯勒势或更高阶 F 项产生贡献)。这些零模可以通过奥里 enti 福尔德分次层上同调群 $H^i_\pm(D, \mathcal{O}_D)$ (满足 $i = 0, 1, 2$) 的维数计数。如果 D 光滑、有效，且满足刚性条件 [15]

$$\dim H^\bullet_+(D, \mathcal{O}_D) = (1, 0, 0)\,, \ \dim H^\bullet_-(D, \mathcal{O}_D) = 0, \tag{37}$$

with $\bullet$ standing for $i = 0, 1, 2$, then [16] Euclidean D3-branes wrapping D generate the non-perturbative superpotential term:

其中 $\bullet$ 代表 $i = 0, 1, 2$ ，那么包裹 D 的 [16] 个欧氏 D3 膜会生成如下非微扰超势项:

$$W_{\mathrm{ED3}|D,\,\mathrm{rigid}} = \mathcal{A}(z_i, z_{D3}, z_{D7}, \tau)\, e^{-2\pi c^a T_a}. \tag{38}$$

The prefactor $\mathcal{A}(z_i, z_{D3}, z_{D7}, \tau)$ is a one-loop Pfaffian that can depend on the complex structure moduli, the axiodilaton, and the positions z_{D3} and z_{D7} of any spacetime-filling D3-branes (see, e.g., [44-47]) and D7-branes [48]. Rigidity of D implies that $\mathcal{A}(z_i, z_{D3}, z_{D7}, \tau)$ is not identically vanishing, though it is generally a section of a nontrivial bundle on moduli space, with zeros on certain subloci - for example, if a spacetime-filling D3-brane coincides with D [49].

前置因子 $\mathcal{A}(z_i, z_{D3}, z_{D7}, \tau)$ 是一圈 Pfaffian，它可以依赖于复结构模、轴子 dilaton 以及所有填充时空的 D3 膜 z_{D3} 和 D7 膜 z_{D7} 的位置 (例如参见 [44-47] 和 [48])。D 的刚性意味着 $\mathcal{A}(z_i, z_{D3}, z_{D7}, \tau)$ 不恒为零，但它一般是模空间上一个非平凡丛的截面，在某些子轨迹上为零——例如当填充时空的 D3 膜与 D 重合时 [49]。

If a smooth divisor D fulfilling (37) obeys the further condition that its uplift to F-theory [17] has trivial intermediate Jacobian, we call D pure rigid, and its Pfaffian has no dependence on z_i, z_{D7} , or τ : the Pfaffian of a pure rigid divisor is a section of the trivial bundle over the moduli space of the associated fourfold [55].

如果满足 (37) 的光滑除子 D 进一步满足其提升到 F 理论 [17] 后中间雅可比为平凡的条件，我们称 D 为纯刚性除子，其 Pfaffian 不依赖于 z_i, z_{D7} 或 τ : 纯刚性除子的 Pfaffian 是对应四维流形模空间上平凡丛的截面 [55]。

Thus, a pure rigid divisor in a compactification that contains no spacetime-filling D3-branes generates a simple exponential:

因此，在不含填充时空 D3 膜的紧致化中，纯刚性除子会生成一个简单的指数项:

$$W_{\mathrm{ED3}|D,\,\text{pure rigid}} = \mathcal{A} e^{-2\pi c^a T_a}, \tag{39}$$

with $\mathcal{A}$ a constant.

其中 $\mathcal{A}$ 是常数。

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[15] Beware that in some settings D is called rigid if it fulfills the weaker condition $\dim H^2(D, \mathcal{O}_D) = 0$.

[15] 注意，在某些场景中，只要满足更弱的条件 $\dim H^2(D, \mathcal{O}_D) = 0$ 就被称为刚性 D 。

[16] The rigidity condition (37) alone is neither necessary nor sufficient for a non-perturbative superpotential contribution on an effective divisor D . If D is smooth, then (37) is indeed sufficient [38]. For certain singular D, (37) is not necessary, but a corrected sufficient condition that accounts for the singular loci is applicable [39]. For certain other D that do not fulfill (37) by fault of having $\dim H^2(D, \mathcal{O}_D) > 0$, rigidification by flux can lead to a superpotential term: Euclidean D3-branes wrapping such a D and bearing magnetic flux can contribute to W (see, e.g.,[40-43]). However, in the examples given below, we will be able to consistently neglect contributions from singular divisors or divisors rigidified by flux.

[16] 仅靠刚性条件 (37) 对于有效除子 D 的非微扰超势贡献而言既不充分也不必要。如果 D 光滑，那么 (37) 确实是充分条件 [38]。对于某些奇异 D, (37)，(37) 不是必要的，但存在一个考虑了奇异轨迹的修正充分条件 [39]。对于另一些因满足 $\dim H^2(D, \mathcal{O}_D) > 0$ 而不满足 (37) 的 D，通量产生的刚性化也可以带来超势项: 欧几里得 D3 膜包裹这种 D 并携带磁通量，可以对 W 产生贡献 (例如参见 [40-43])。但在下文的例子中，我们可以始终忽略奇异除子或经通量刚性化的除子的贡献。

[17] For reviews on F-theory, see [50,51], and for relations between Euclidean D3-branes and Euclidean M5-branes, see, e.g., [52]. Euclidean M5-branes are considered from the perspective of the heterotic dual

in [53]. A comprehensive review of D-brane instantons, including many phenomena we have omitted here, appears in [54].

[17] 关于 F 理论的综述参见 [50,51]，关于欧几里得 D3 膜与欧几里得 M5 膜的关系例如参见 [52]。文献 [53] 从杂化对偶的视角研究了欧几里得 M5 膜。文献 [54] 对 D 膜瞬子 (包括本文省略的诸多现象) 给出了全面综述。

The gaugino condensate superpotential is rather analogous to W_{ED3} . Consider a stack of D7-branes that wrap an effective divisor $D = c^a D_a$ and generate an $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with gauge group G and with $h^{0,2}(D) + h^{0,1}(D)$ chiral multiplets charged in the adjoint of G . If $h^{0,2}(D) = h^{0,1}(D) = 0$, then the theory is pure super-Yang-Mills, and at low energies it generates a gaugino condensate superpotential:

金尼诺凝聚超势与 W_{ED3} 非常类似。考虑一堆 D7 膜，它们包裹有效除子 $D = c^a D_a$ ，生成具有规范群 G 、包含带电属于 G 伴随表示的 $h^{0,2}(D) + h^{0,1}(D)$ 手征多重态的 $\mathcal{N} = 1$ 超对称杨-米尔斯理论。如果 $h^{0,2}(D) = h^{0,1}(D) = 0$ ，该理论就是纯超杨-米尔斯，在低能下会生成金尼诺凝聚超势:

$$W_{\lambda\lambda|D} = \mathcal{A}(z_i, z_{D3}, \tau)\, e^{-2\pi c^a T_a / c(G)}, \tag{40}$$

where $c(G)$ is the dual Coxeter number of G . The Pfaffian now has the interpretation of resulting from a threshold correction to the gauge coupling. In particular, the dependence on z_{D3} occurs because strings stretched between the D7-brane stack and a D3-brane produce chiral multiplets charged in the fundamental representation of G , and whose masses affect the low-energy condensate.

其中 $c(G)$ 是 G 的对偶考克斯特数。此时 Pfaffian 可解释为规范耦合的阈修正。具体来说，对 z_{D3} 的依赖来源于: 伸展在 D7 膜堆和 D3 膜之间的弦会产生带电属于 G 基础表示的手征多重态，其质量会影响低能凝聚。

The full superpotential thus takes the form:

因此完整的超势形式为:

$$W = W_{\mathrm{flux}}(\tau, z_i) + W_{\mathrm{np}}(\tau, z_i, T_a), \tag{41}$$

with

其中

$$W_{\mathrm{np}} = W_{\mathrm{ED3}} + W_{\lambda\lambda} + W_{\mathrm{ED}(-1)}. \tag{42}$$

Here W_{ED3} denotes the sum of all Euclidean D3-brane terms, $W_{\lambda\lambda}$ is the sum of all gaugino condensate terms, and $W_{\mathrm{ED}(-1)}$ is generated by Euclidean D(-1)-branes, i.e., D-instantons, and obeys

此处 W_{ED3} 表示所有欧几里得 D3 膜项的总和，$W_{\lambda\lambda}$ 是所有金微子凝聚项的总和，$W_{\mathrm{ED}(-1)}$ 由欧几里得 D(-1) 膜 (即 D 瞬子) 产生，满足

$$W_{\mathrm{ED}(-1)} = \mathcal{O}\left(e^{-\pi\tau}\right). \tag{43}$$

Because W_{ED3} and $W_{\lambda\lambda}$ are exponential in four-cycle volumes, and $W_{\mathrm{ED}(-1)}$ is exponential in τ , we conclude that to all perturbative orders in the α' and g_s expansions, the superpotential is given by

由于 W_{ED3} 和 $W_{\lambda\lambda}$ 关于四周期体积是指数形式，$W_{\mathrm{ED}(-1)}$ 关于 τ 也是指数形式，因此我们得出结论：在 α' 和 g_s 展开中，到所有微扰阶数下，超势由下式给出

$$W \approx W_{\mathrm{flux}}, \tag{44}$$

up to corrections that are exponentially small when four-cycle volumes T_a are large and the string coupling $\mathrm{Im}(\tau)$ is weak.

修正项除外，当四周期体积 T_a 很大且弦耦合 $\mathrm{Im}(\tau)$ 很弱时，这些修正项呈指数小量。

Kähler Potential

凯勒势

Tree Level

树图阶

At the sphere level, the moduli space of closed string fields in an O3/O7 orientifold of a Calabi-Yau threefold X_6 factorizes into the complex structure, Kähler, and axiodilaton moduli spaces:

在球面阶，卡拉比-丘三维流形 O3/O7 定向形 X_6 中的闭弦场模空间可分解为复结构、凯勒和轴子 dilaton 模空间:

$$\mathcal{M} = \mathcal{M}_{\mathrm{cs}}(X_6) \times \mathcal{M}_{\mathrm{K}}(X_6) \times \mathcal{M}_\tau. \tag{45}$$

Therefore, the metric for the moduli is block diagonal, and the Kähler potential splits into dilaton, complex structure, and Kähler moduli terms [19]:

因此，模空间的度规是分块对角的，凯勒势可拆分为 dilaton、复结构和凯勒模项 [19]:

$$K_{\mathrm{tree}} = -\ln\left(-i\left(\tau-\bar{\tau}\right)\right) - \ln\left(-i\int_{X_6}\Omega\left(z_i\right)\wedge\overline{\Omega}\left(\bar{z}_i\right)\right) - 2\ln\left(\mathcal{V}\left(T_a,\bar{T}_a\right)\right). \tag{46}$$

The holomorphic $(3,0)$ -form Ω encodes the dependence on complex structure moduli, as can be seen by expanding in a basis of three forms. At large complex structure (LCS), we have:

全纯 $(3,0)$ 形式 Ω 包含了对复结构模的依赖，这一点可以通过在三形式基下展开看出。在大复结构 (LCS) 极限下，我们有:

$$\int_{X_6} \Omega \wedge \overline{\Omega} = \vec{\Pi}^{\dagger} \cdot \sum \cdot \vec{\Pi}, \tag{47}$$

in terms of the periods $\vec{\Pi}$ defined in (28) and the symplectic matrix $\sum$ defined in (30).

其中各项基于 (28) 式定义的周期 $\vec{\Pi}$ 和 (30) 式定义的辛矩阵 $\sum$ 。

To understand the dependence of K_{tree} on the Kähler moduli, we recall from (32) that the complexified four-cycle volumes are $T_a = \tau_a + i\theta_a$, with axion fields θ_a appearing from the compactification of the Ramond-Ramond four-form $C_4, \theta_a = \int_{D_a} C_4$. As we explained in section "Non-perturbative Superpotential", the axions θ_a enjoy PQ symmetries (33) inherited from the gauge symmetry of C_4 . In combination with holomorphy of W , the PQ symmetries (33) forbid the T_a from appearing in W at any perturbative order. But the Kähler potential is not holomorphic, and so K_{tree} and K_{pert} do depend on the τ_a , but not on the θ_a .

为了理解 K_{tree} 对凯勒模的依赖，我们从 (32) 式回顾，复化四周期体积为 $T_a = \tau_a + i\theta_a$ ，其中轴子场 θ_a 来自拉蒙德-拉蒙德四形式 $C_4, \theta_a = \int_{D_a} C_4$ 的紧致化。正如我们在“非微扰超势”一节中说明的，轴子 θ_a 具有继承自 C_4 规范对称性的 PQ 对称性 (33)。结合 W 的全纯性，PQ 对称性 (33) 禁止 T_a 在任意微扰阶出现在 W 中。但凯勒势不是全纯的，因此 K_{tree} 和 K_{pert} 确实依赖于 τ_a ，但不依赖于 θ_a 。

We introduce a basis $\{\omega_a\}$ of $H^2(X, \mathbb{Z})$ and write the Kähler form as

我们引入 $H^2(X, \mathbb{Z})$ 的一组基 $\{\omega_a\}$ ，将凯勒形式写为

$$J = t^a \omega_a \tag{48}$$

where the Kähler parameters t^a measure the volumes of two cycles. Then, the volume $\mathcal{V}$ of X_6 can be written:

其中凯勒参数 t^a 度量二周期的体积。于是 X_6 的体积 $\mathcal{V}$ 可以写为:

$$\mathcal{V} = \frac{1}{6} \int_{X_6} J \wedge J \wedge J = \frac{1}{6} \kappa_{abc} t^a t^b t^c, \tag{49}$$

where

其中

$$\kappa_{abc} = \int_X \omega_a \wedge \omega_b \wedge \omega_c \tag{50}$$

are the triple intersection numbers. The four-cycle volumes τ_a are then given by

是三重相交数。接下来四周期体积 τ_a 由下式给出

$$\tau_a = \frac{\partial \mathcal{V}}{\partial t^a} = \frac{1}{2}\kappa_{abc}t^b t^c. \tag{51}$$

Note that $\mathcal{V}$ is written explicitly as a function of the t^a , but the Kähler coordinates are $T_a = \tau_a + i\theta_a$. The relation between t^a and τ_a is not in general analytically invertible, and so the dependence of K on τ_a is usually only implicit, via $K(\tau_a) = K\left(t^b(\tau_a)\right)$.

注意 $\mathcal{V}$ 已经显式写为 t^a 的函数，但凯勒坐标是 $T_a = \tau_a + i\theta_a$ 。t^a 和 τ_a 之间的关系一般不存在解析逆，因此 K 对 τ_a 的依赖通常仅通过 $K(\tau_a) = K\left(t^b(\tau_a)\right)$ 隐式给出。

Even though the dependence of K_{tree} on the τ_a is complicated and model-dependent, its dependence on $\mathcal{V}$ is simple. Under the scaling (16),(17) with $v = 1/2$,

尽管 K_{tree} 对 τ_a 的依赖复杂且依赖具体模型，但它对 $\mathcal{V}$ 的依赖十分简单。在满足 $v = 1/2$ 的标度变换 (16)、(17) 下，

$$\tau_a \to \lambda\tau_a \tag{52}$$

we have:

我们有:

$$\mathcal{V} \to \lambda^{3/2}\mathcal{V},\ K_{\text{tree}} \to K_{\text{tree}} - 3\ln\lambda. \tag{53}$$

By taking derivatives with respect to λ and evaluating at $\lambda = 1$, we derive the identity:

通过对 λ 求导并在 $\lambda = 1$ 处求值，我们推导出等式:

$$K^{a\bar{b}}_{\text{tree}} K^{\text{tree}}_a K^{\text{tree}}_{\bar{b}} = 3. \tag{54}$$

This powerful result is known as the no-scale property [56].

这一强有力的结果被称为无标度性质 [56]。

A key consequence of no-scale structure is that the negative term $-3|W|^2$ in the scalar potential (21) is precisely cancelled by the term $K^{a\bar{b}}_{\text{tree}} K^{\text{tree}}_a K^{\text{tree}}_{\bar{b}} |W|^2$. Thus, if the superpotential is independent of the Kähler moduli - which is true at tree level, cf. W_{tree} in (27) - the scalar potential reduces to

无标度结构的一个核心结论是，标量势 (21) 中的负项 $-3|W|^2$ 被项 $K^{a\bar{b}}_{\text{tree}} K^{\text{tree}}_a K^{\text{tree}}_{\bar{b}} |W|^2$ 完全抵消。因此，如果超势不依赖于凯勒模——这在树图阶成立，参见 (27) 中的 W_{tree} ——则标量势退化为

$$V_{\text{tree}} = V_{\text{no-scale}} = e^K K^{M\bar{N}} D_M W \bar{D}_{\bar{N}} \bar{W}, \tag{55}$$

where now $M, N = 1, \ldots, h^{2,1} + 1$, corresponding to the complex structure moduli z_i and the dilaton τ , and everything on the right-hand side of (55) is understood to be evaluated using W_{tree} and K_{tree} .

其中此处 $M, N = 1, \ldots, h^{2,1} + 1$ 对应复结构模 z_i 和 dilaton(dilation 场) τ ，且 (55) 右侧的所有项均需理解为使用 W_{tree} 和 K_{tree} 计算得出。

The importance of this result cannot be overemphasized. Some of its implications are:

该结果的重要性怎么强调都不为过，它的部分结论如下:

- The tree-level scalar potential $V_{\text{no-scale}}$ is positive-definite, and so the minima in the z_i and τ directions have vanishing F-terms for these fields: $D_M W = 0$.

- 树级标量势 $V_{\text{no-scale}}$ 是正定的，因此 z_i 和 τ 方向的极小值处这些场的 F 项为零: $D_M W = 0$ 。

- Supersymmetry is broken in the Kähler moduli directions, since $D_{T_a} W$ is arbitrary in the minimum of $V_{\text{no-scale}}$.

- 凯勒模方向上超对称破缺，因为在 $V_{\text{no-scale}}$ 的极小值处 $D_{T_a} W$ 是任意的。

- The value of $V_{\text{no-scale}}$ is zero at the minimum, which means that the cosmological constant vanishes, even though supersymmetry is broken (by the F-terms $D_{T_a} W$).

- 极小值处 $V_{\text{no-scale}}$ 的值为零，这意味着即使 (被 F 项 $D_{T_a} W$) 破缺了超对称，宇宙学常数仍然为零。

- The presence of any source of positive energy will drive a runaway to large volume: recall that $e^K \propto \mathcal{V}^{-2}$ times higher-order terms in the $1/\mathcal{V}$ expansion.

- 任何正能源的存在都会导致大体积 runaway(跑 away) 行为: 还记得 $e^K \propto \mathcal{V}^{-2}$ 乘以 $1/\mathcal{V}$ 展开中的高阶项吗?

- The Kähler moduli T_a are not stabilized by $V_{\text{no-scale}}$. Thus, stabilization of the T_a is possible only once quantum corrections to the scalar potential - through one or more of $W_{\text{np}}, K_{\text{pert}}$, and K_{np} - impact the vacuum structure.

- 凯勒模 T_a 无法被 $V_{\text{no-scale}}$ 稳定。因此，只有对标量势的量子修正——通过 $W_{\text{np}}, K_{\text{pert}}$ 和 K_{np} 中的一种或多种——改变真空结构后，T_a 才有可能被稳定。

Perturbative Corrections

微扰修正

The Kähler potential beyond tree level is the least understood of the quantities that are important in moduli stabilization. The tree-level expression K_{tree} in (46) is the dominant contribution to the kinetic terms, but, due to the no-scale property (54), the scalar potential computed with $K = K_{\text{tree}}$ and $W = W_{\text{tree}} = W_{\text{flux}}$ vanishes. Thus, the leading contributions to V will come from higher-order perturbative (or non-perturbative) corrections to K , or from the non-perturbative superpotential.

树图阶之外的凯勒势是模稳定中最重要但目前研究最不充分的物理量之一。(46) 中的树图阶表达式 K_{tree} 是动力学项的主导贡献，但由于无标度性质 (54)，使用 $K = K_{\text{tree}}$ 和 $W = W_{\text{tree}} = W_{\text{flux}}$ 计算得到的标量势为零。因此，V 的主导贡献来自对 K 的高阶微扰 (或非微扰) 修正，或是来自非微扰超势。

We have seen in section ”Scalings and Perturbative Expansions” that the Kähler potential receives correction order by order in perturbation theory in both the string loop and α' expansions. The scaling properties (19) serve to organize these corrections: the tree-level expression K_{tree} given in (46) satisfies the $(m, n) = (0, 0)$ scaling, and for general values of m, n we use (19) to write the expression for the Kähler potential to all orders in perturbation theory as

我们已经在“标度与微扰展开”一节中看到，凯勒势会在弦圈和 α' 展开中逐阶接受微扰修正。标度性质 (19) 可用于整理这些修正:(46) 给出的树图阶表达式 K_{tree} 满足 $(m, n) = (0, 0)$ 标度，对于一般的 m, n 值，我们可以利用 (19)，将所有阶微扰下的凯勒势表达式写为

$$e^{-K/3} = (\operatorname{Im}\tau)^{1/3}\mathcal{V}^{2/3}\sum_{m,n} A^{(m,n)}\left(\frac{1}{\operatorname{Im}\tau}\right)^{n}\left[\frac{(\operatorname{Im}\tau)^{1/2}}{\mathcal{V}^{1/3}}\right]^{m}. \tag{56}$$

The coefficients $A^{(m,n)}$ depend on scale-invariant combinations of the fields, such as the complex structure moduli z_i or ratios of Kähler moduli.

系数 $A^{(m,n)}$ 依赖于场的标度不变组合，例如复结构模 z_i 或凯勒模的比值。

In terms of the tree-level superpotential $W_0 := \langle W_{\text{flux}} \rangle$, cf. (27), we write the scalar potential at order m, n as

参照 (27)，我们可以用树图阶超势 $W_0 := \langle W_{\text{flux}} \rangle$ 将 m, n 阶的标量势写为

$$V^{(m,n)} = B^{(m,n)}\left(\frac{1}{\operatorname{Im}\tau}\right)^{n}\left[\frac{(\operatorname{Im}\tau)^{1/2}}{\mathcal{V}^{1/3}}\right]^{m}\frac{|W_0|^2}{\mathcal{V}^2\operatorname{Im}\tau}, \tag{57}$$

where $B^{(m,n)}$ are scale-invariant combinations of the fields.

其中 $B^{(m,n)}$ 是场的标度不变组合。

The expression (21) for the scalar potential in terms of K and W is the most general possibility for an $\mathcal{N} = 1$ supersymmetric Lagrangian in which each term has at most two spacetime derivatives of the fields. However, in general there are higher-derivative corrections to the Lagrangian, for example, involving higher powers of the curvature. By supersymmetry, such higher-derivative corrections can also affect the structure of the scalar potential, and need to be considered.

用 K 和 W 表示标量势的表达式 (21)，是 $\mathcal{N}=1$ 超对称拉格朗日量最一般的形式，该形式中每个项最多包含两个场的时空导数。但一般而言拉格朗日量会存在高导数修正，例如涉及曲率的高次幂的修正。根据超对称，这类高导数修正也会影响标量势的结构，因此需要纳入考虑。

In string theory, these corrections to the scalar potential include higher powers of the fluxes G_3 , corresponding to higher powers of W_0 , which in turn reflect higher-order superspace covariant derivatives, i.e., F-terms. The scaling analysis easily captures these higher powers of W_0 by writing:

在弦论中，这些对标量势的修正包含流 G_3 的高次幂，对应 W_0 的高次幂，而后者又反映了超空间协变导数即 F 项的高阶。标度分析可以很容易地通过如下写法涵盖 W_0 的这些高次幂:

$$B^{(m,n)} = B^{(m,n,r)}\left(\frac{|W_0|^2}{\mathcal{V}^{2/3}\,\mathrm{Im}\,\tau}\right)^{r-1}, \quad m = p + r - 1 \tag{58}$$

where $B^{(m,n,r)}$ is a scale-invariant function of the fields that is independent of W_0 . The quantity $\varepsilon := |W_0|^2/\left(\mathcal{V}^{2/3}\,\mathrm{Im}\,\tau\right)$ is a scale-invariant function of the fields proportional to $\left(gF/M^2\right)^2 \simeq \left(m_{3/2}/M\right)^2$, where F is the auxiliary field that breaks supersymmetry; M is the cutoff scale, here identified with the Kaluza-Klein scale; $m_{3/2}$ is the gravitino mass; and g is the coupling between heavy Kaluza-Klein states and light modes. Then, $\varepsilon \simeq \left(m_{3/2}/M\right)^2$ is a natural small parameter, as required for the validity of the EFT at scales below the cutoff M .

其中 $B^{(m,n,r)}$ 是不依赖于 W_0 的场的标度不变函数。量 $\varepsilon := |W_0|^2/\left(\mathcal{V}^{2/3}\,\mathrm{Im}\,\tau\right)$ 是正比于 $\left(gF/M^2\right)^2 \simeq \left(m_{3/2}/M\right)^2$ 的场的标度不变函数, 其中 F 是破缺超对称的辅助场; M 是截断标度, 这里等同于卡鲁扎-克莱因标度; $m_{3/2}$ 是引力微子质量; g 是重卡鲁扎-克莱因态与轻模式之间的耦合。那么 $\varepsilon \simeq \left(m_{3/2}/M\right)^2$ 是一个自然的小参数，这是有效场论在截断 M 以下能标成立所要求的。

The scalar potential at each given order (m,n,r) in the α' , string loop, and F-term expansions, with small parameters $1/\mathcal{V}, 1/\,\mathrm{Im}\,\tau$, and ε , respectively, can then be written as

在 α' 展开、弦圈展开和 F 项展开中，给定 (m,n,r) 阶的标量势，分别以小参数 $1/\mathcal{V}, 1/\,\mathrm{Im}\,\tau$ 和 ε ，可写为

$$V^{(m,n,r)} = \frac{B^{(m,n,r)}}{\mathcal{V}^{4/3}}\left(\frac{1}{\mathrm{Im}\,\tau}\right)^n\left[\frac{(\mathrm{Im}\,\tau)^{1/2}}{\mathcal{V}^{1/3}}\right]^m\left(\frac{|W_0|^2}{\mathcal{V}^{2/3}\,\mathrm{Im}\,\tau}\right)^r. \tag{59}$$

The leading-order terms can be classified as follows:

领头阶项可以分类如下:

- $m = n = 0, r = 1$. This is the standard tree-level potential:

- $m = n = 0, r = 1$ 。这是标准的树图标量势:

$$V^{(0,0,1)} = \frac{B^{(0,0,1)}|W_0|^2}{\mathrm{Im}\,\tau\,\mathcal{V}^2}, \quad \text{no-scale} \Rightarrow B^{(0,0,1)} = 0. \tag{60}$$

- $m=1, n=0, r=1$. This α'^1 term may in principle appear, and the corresponding scalar potential

> - $m=1, n=0, r=1$ 。原则上该 α'^1 项可以存在，对应的标量势

$$V^{(1,0,1)} = \frac{B^{(1,0,1)}|W_0|^2}{\sqrt{\operatorname{Im}\tau}\mathcal{V}^{7/3}}, \tag{61}$$

could be dominant in a $1/\mathcal{V}$ expansion, because the tree-level potential $V^{(0,0,1)}$ vanishes. However, general dimensional analysis arguments show that $B^{(1,0,1)}=0$ [12]. Even though corrections that lead to $\mathcal{V}^{-7/3}$ behavior for the scalar potential have not yet been identified, it is an open question whether they may occur at higher string loops, i.e., for $(m,n,r)=(1,n,1)$ with $n>0$.

> 在 $1/\mathcal{V}$ 展开中可以占主导，因为树级势 $V^{(0,0,1)}$ 为零。然而，一般量纲分析论证表明 $B^{(1,0,1)}=0$ [12]。尽管目前尚未找到能导致标量势呈现 $\mathcal{V}^{-7/3}$ 行为的修正，这类修正是否会出现在更高弦圈阶 (即满足 $n>0$ 条件的 $(m,n,r)=(1,n,1)$) 仍是一个开放问题。

- $m=2, n=2, r=1$. This is an α'^2 correction at loop order in the string expansion, either as one-loop open strings (the open string loop counting parameter is $n-1$) or as tree-level exchange of Kaluza-Klein states among D-branes [57-61] (see also [62]). However, since the terms in $e^{-K/3}$ proportional to $A^{(2,n)}$ are independent of $\mathcal{V}$ for all n , cf. (56), the associated contribution to the scalar potential vanishes, corresponding to what has been called extended no-scale structure [59, 60] . It follows that $B^{(2,2,1)}=0$.

> - $m=2, n=2, r=1$ 。这是弦展开中圈阶的 α'^2 修正，既可以是一圈开弦 (开弦圈计数参数为 $n-1$)，也可以是 D 膜之间卡鲁扎-克莱因态的树级交换 [57-61](另见 [62])。但由于 $e^{-K/3}$ 中正比于 $A^{(2,n)}$ 的项对所有 n 都不依赖 $\mathcal{V}$ ，参见式 (56)，其对标量势的贡献为零，对应所谓的推广无标度结构 [59, 60]，由此可得 $B^{(2,2,1)}=0$ 。

$$V^{(2,2,1)} = \frac{B^{(2,2,1)}|W_0|^2}{(\operatorname{Im}\tau)^2\mathcal{V}^{8/3}}, \quad \text{extended no-scale} \Rightarrow B^{(2,2,1)}=0. \tag{62}$$

The same argument applies to any other allowed values for n and r for fixed $m=2$, i.e., $B^{(2,n,r)}=0$.

> 相同论证适用于固定 $m=2$ 时 n 和 r 的任意其他允许取值，即 $B^{(2,n,r)}=0$ 。

- $m=3, n=0, r=1$. These are string tree-level, α'^3 corrections that have been explicitly calculated [15] and come from the ten-dimensional $\mathcal{R}^4, \mathcal{R}^3|G_3|^2$, and $\mathcal{R}^2(\nabla G_3)^2$ terms:

> - $m=3, n=0, r=1$ 。这些是弦树级的 α'^3 修正，已被显式计算 [15]，来自十维的 $\mathcal{R}^4, \mathcal{R}^3|G_3|^2$ 和 $\mathcal{R}^2(\nabla G_3)^2$ 项:

$$V^{(3,0,1)} = \frac{B^{(3,0,1)}\sqrt{\operatorname{Im}\tau}|W_0|^2}{\mathcal{V}^3}. \tag{63}$$

Explicit calculations determine the coefficient $B^{(3,0,1)}$ to be proportional to the Euler number of the original Calabi-Yau manifold. This correction is the best understood: it is an $\mathcal{N}=2$ supersymmetric correction that is present even without orientifolding [63-66]. This result is expected to receive $\mathcal{N}=1$ corrections.

显式计算表明系数 $B^{(3,0,1)}$ 正比于原始卡拉比-丘流形的欧拉数。该修正是目前理解最透彻的: 它是 $\mathcal{N}=2$ 超对称修正，即便不存在 orientifold 投影也存在 [63-66]，且人们认为该结果会受到 $\mathcal{N}=1$ 修正。

- $m=4, n=2, r=1$. These are α'^4 , open string one-loop and $\mathcal{O}\left(F^2\right)$ corrections:

- $m=4, n=2, r=1$ 。这些是 α'^4 、开弦一圈和 $\mathcal{O}\left(F^2\right)$ 修正:

$$V^{(4,2,1)}=\frac{B^{(4,2,1)}\left|W_0\right|^2}{\operatorname{Im}\tau\mathcal{V}^{10/3}}. \tag{64}$$

These corrections have also been computed in particular cases [57], and are subdominant compared to the α'^3 correction $V^{(3,0,1)}$ when $\mathcal{V}$ is large enough to trust the α' expansion.

这些修正也已在特定情形下算出 [57]，当 $\mathcal{V}$ 足够大、α' 展开可信时，它们比 α'^3 修正 $V^{(3,0,1)}$ 更次要。

- $m=3, n=0, r=2$. These are string tree-level, α'^3 corrections at order $\mathcal{O}\left(F^4\right)$ in auxiliary fields:

- $m=3, n=0, r=2$ 。这些是弦树级下，辅助场 $\mathcal{O}\left(F^4\right)$ 阶的 α'^3 修正:

$$V^{(3,0,2)}=\frac{B^{(3,0,2)}\left|W_0\right|^4}{\sqrt{\operatorname{Im}\tau}\mathcal{V}^{11/3}}. \tag{65}$$

We could continue with terms that depend on higher powers of the string coupling and inverse volume, but typically these will be subdominant at large volume and weak coupling [18] . Moreover, we have considered only a power-law expansion of $e^{-K/3}$, and in general the series expansion may include logarithmic terms. Such corrections may result from loops of light fields rather than of heavy Kaluza-Klein modes. Corrections of this type were recently considered in [73-78].

我们还可以继续讨论依赖更高次幂弦耦合和逆体积的项，但通常这类项在大体积弱耦合下 [18] 是更次要的。此外，我们这里仅考虑了 $e^{-K/3}$ 的幂律展开，一般而言级数展开还可以包含对数项。这类修正通常来自轻场的圈，而非重卡鲁扎-克莱因模的圈。这类修正最近已有研究，见文献 [73-78]。

The expansion (59) we have given for the scalar potential is in terms of powers of $1/\mathcal{V}, 1/\operatorname{Im}\tau$, and ε . However, once D-branes are incorporated, there is a new expansion parameter: the gauge coupling in the D-brane theory, which is inversely proportional to the volume of the cycle wrapped by the D-brane.

我们给出的标量势展开式 (59) 是按 $1/\mathcal{V}, 1/\operatorname{Im}\tau$ 和 ε 的幂次展开的。然而，一旦引入 D 膜，就会出现一个新的展开参数:D 膜理论中的规范耦合，它与 D 膜所卷绕闭链的体积成反比。

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[18] A variety of techniques have been used to extract information about corrections to the Kähler potential, including explicit string worldsheet calculations and M-theory and F-theory analyses. Both α'^2 and α'^3 corrections have been obtained, consistent with the scaling behavior outlined above. Some such contributions shift the associated coefficient $B^{(m,n,r)}$, while others can be absorbed by field redefinitions. See for instance [67-72].

[18] 人们已经使用多种方法来提取卡勒势能修正的相关信息，包括显式弦世界面计算以及 M 理论和 F 理论分析。α'^2 修正和 α'^3 修正均已得到，且与上文概述的标度行为一致。部分这类修正会改变关联系数 $B^{(m,n,r)}$，其余修正则可通过场重定义吸收。参见例如文献 [67-72]。

Thus, each of the Kähler moduli may provide an extra expansion parameter, just as in section "Non-perturbative Superpotential" the superpotential received non-perturbative corrections in an expansion in cycle volumes.

因此，每个卡勒模都可以提供一个额外的展开参数，就像“非微扰超势”一节中超势获得了按闭链体积展开的非微扰修正一样。

Vacua in Type IIB Compactifications

IIB 型紧致化中的真空

We have now introduced enough of the structure of type IIB flux compactifications on O3/O7 orientifolds to be able to detail the construction of isolated vacua, i.e., solutions with stabilized moduli.

我们现在已经充分介绍了 O3/O7 定向形上 IIB 型通量紧致化的结构，接下来可以详细介绍孤立真空也就是模稳定的解的构造了。

KKLT

KKLT

We will begin with the KKLT scenario, proposed 20 years ago by Kachru, Kallosh, Linde, and Trivedi [79].

我们将从 KKLT 场景开始讲起，它是 20 年前由 Kachru、Kallosh、Linde 和 Trivedi 提出的 [79]。

At leading order in the α' and g_s expansions, the effective theory of an O3/O7 orientifold flux compactification is described by

在 α' 和 g_s 展开的领头阶，O3/O7 定向轨形通量紧致化的有效理论可描述为

$$W \approx W_{\text{flux}}\,(z_i, \tau)\,,\; K \approx K_{\text{tree}}\,, \tag{66}$$

where W_{flux} and K_{tree} are given in given in (27) and (46), respectively. In such theories there typically exist points $z_i^\star, \tau^\star$ in the joint complex structure and axiodilaton moduli space at which the F-terms of these fields vanish:

其中 W_{flux} 和 K_{tree} 分别由 (27) 和 (46) 给出。在这类理论中，复结构和轴 dilaton 联合模空间里通常存在满足这些场的 F 项为零的点 $z_i^\star, \tau^\star$:

$$D_{z_i} W\big|_{z_i^\star,\tau^\star} = D_\tau W\big|_{z_i^\star,\tau^\star} = 0, \tag{67}$$

and the complex structure moduli and axiodilaton are stabilized by supersymmetric mass terms. We define the expectation value of the flux superpotential:

复结构模和轴 dilaton 由超对称质量项稳定。我们定义通量超势的期望值:

$$W_0 := \langle W_{\text{flux}} \rangle \equiv W_{\text{flux}}\left(z_i^\star, \tau^\star\right), \tag{68}$$

where the brackets denote evaluation on the configuration $z_i^\star, \tau^\star$ at which the complex structure moduli and axiodilaton are stabilized.

其中方括号表示在复结构模和轴 dilaton 已稳定的构型 $z_i^\star, \tau^\star$ 上求值。

The F-terms of the Kähler moduli T_a , again at leading order in α' and g_s , read:

Kähler 模 T_a 的 F 项，同样在 α' 和 g_s 的领头阶，表达式为:

$$D_{T_a} W\big|_{z_i^\star,\tau^\star} \approx W_0 \partial_{T_a} K_{\text{tree}}\Big| \tag{69}$$

and so W_0 is the order parameter measuring supersymmetry breaking. Except in the highly nongeneric situation where $W_0 = 0$, the F-terms (69) do not vanish anywhere inside the moduli space, and in the classical theory at leading order in α' , supersymmetry is broken by the Kähler moduli F-terms.

因此 W_0 是衡量超对称破缺的序参量。除非出现非常非 generic 的 $W_0 = 0$ 情形，否则式 (69) 的 F 项在模空间内处处非零；在领头阶 α' 的经典理论中，超对称性由 Kähler 模的 F 项破缺。

The first key idea of the KKLT scenario is that if W_0 is exponentially small, then the correspondingly small breaking of supersymmetry by fluxes can be compensated by a non-perturbative superpotential term, leading to supersymmetric stabilization of all moduli in an AdS$_4$ vacuum, in a parameter regime where computational control is possible.

KKLT 场景的第一个核心思想是: 如果 W_0 是指数小量，那么通量带来的相应小超对称破缺可以被非微扰超势项抵消，最终在计算可控的参数区域中得到一个 AdS$_4$ 真空，实现所有模的超对称稳定。

The second key idea is that if the complex structure moduli are stabilized near a point in moduli space where a conifold singularity occurs, the compactification can contain a strongly warped region, and the presence of an anti-D3-brane in this region can lead to controllable breaking of supersymmetry.

第二个核心思想是: 如果复结构模稳定在模空间中锥奇点附近的位置，紧致化可以包含一个强翘曲区域；该区域中存在的反 D3 膜可以带来可控制的超对称破缺。

We will first discuss supersymmetric stabilization, deferring a treatment of anti-D3-brane supersymmetry breaking to section "Anti-D3-Branes". For notational simplicity, we will illustrate the stabilization idea in a hypothetical example with $h^{1,1}_{+} = 1$. Explicit models in actual Calabi-Yau orientifolds, whose only important difference from the toy model is that they have multiple Kähler moduli, are presented in section "Explicit Constructions", following [80].

我们先讨论超对称稳定，将反 D3 膜超对称破缺的讨论留到“反 D3 膜”一节。为简化记号，我们在一个带 $h^{1,1}_{+} = 1$ 的假设示例中阐述稳定的核心思想。遵循文献 [80]，实际 Calabi-Yau 定向轨形的显式模型会在“显式构造”一节给出，这类模型和玩具模型唯一的重要区别是它们存在多个 Kähler 模。

Suppose that X_6 is a Calabi-Yau threefold with $h^{1,1} = 1$ and that $\mathcal{O} = \sigma\Omega_{\mathrm{WS}}(-1)^{F_L}$ is an O3/O7 orientifold involution on X_6 such that $X_6/\mathcal{O}$ has $h^{1,1}_{+} = 1$. We denote by T the Kähler modulus of X_6 , and by D the corresponding effective divisor. The Kähler potential reads:

假设 X_6 是带有 $h^{1,1} = 1$ 的 Calabi-Yau 三维空间，$\mathcal{O} = \sigma\Omega_{\mathrm{WS}}(-1)^{F_L}$ 是定义在 X_6 上的 O3/O7 定向轨形对合，且满足 $X_6/\mathcal{O}$ 具有 $h^{1,1}_{+} = 1$ 。我们记 X_6 的 Kähler 模为 T ，对应有效除子为 D 。Kähler 势形式为:

$$K \approx K_{\text{tree}} = -3\log\left(T + \bar{T}\right), \tag{70}$$

and if D supports a pure $\mathcal{N} = 1$ super-Yang-Mills theory on D7-branes, with gauge group G , the superpotential is

若 D 在 D7 膜上支持纯 $\mathcal{N} = 1$ 超杨-米尔斯理论，且规范群为 G ，则超势为

$$W = W_0 + \mathcal{A}\left(z_i, \tau\right) e^{-2\pi T/c(G)} + \ldots \tag{71}$$

If $|W_0| \ll 1$, the F-term of the Kähler modulus, as computed with $K = K_{\text{tree}}$,

如果满足 $|W_0| \ll 1$ ，利用 $K = K_{\text{tree}}$ 计算得到的 Kähler 模的 F 项

$$D_T^{\text{tree}} W = W\partial_T K_{\text{tree}} - \frac{2\pi}{c(G)}\mathcal{A}\left(z_i, \tau\right) e^{-2\pi T/c(G)}, \tag{72}$$

then has a zero at a point $T^{\star}$ inside the moduli space:

会在模空间内的点 $T^{\star}$ 处存在一个零点:

$$\operatorname{Re} T^{\star} = \frac{c(G)}{2\pi}\log\left(|W_0|^{-1}\right) + \ldots, \tag{73}$$

where the omitted corrections, easily obtained from (72), are subleading for $W_0 \ll 1$. At $T = T^{\star}$, the supersymmetry-breaking effects of the flux superpotential W_{flux} and the gaugino condensate superpotential $W_{\lambda\lambda}$ cancel each other.

其中省略的修正项可由式 (72) 直接得到，当 $W_0 \ll$ 远大于 1 时是次领头阶。在 $T = T^\star$ 处，通量超势 W_{flux} 和戈金诺凝聚超势 $W_{\lambda\lambda}$ 的超对称破缺效应相互抵消。

For sufficiently small $|W_0|$, the approximation $K \approx K_{\mathrm{tree}}$ is consistent at $T = T^\star$, and there exists a field configuration very near [19] $z_i^\star, \tau^\star, T^\star$ at which the F-terms of all moduli vanish exactly. The resulting solution is an $\mathcal{N} = 1$ supersymmetric AdS_4 vacuum in which all moduli - the Kähler modulus T , the complex structure moduli z_a , and the axiodilaton $\tau-$ are stabilized.

当 $|W_0|$ 足够小时，近似式 $K \approx K_{\mathrm{tree}}$ 在 $T = T^\star$ 处是自洽的，且存在一个非常接近 [19] $z_i^\star, \tau^\star, T^\star$ 的场位形，其中所有模的 F 项都精确为零。得到的解是一个 $\mathcal{N} = 1$ 超对称 AdS_4 真空，所有模——凯勒模 T 、复结构模 z_a 和轴西塔顿 $\tau-$ ——都被稳定了。

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[19] We discuss the small shifts away from $z_i^\star, \tau^\star, T^\star$ in the example of section ”Explicit Constructions”.

[19] 我们会在“显式构造”一节的例子中讨论偏离 $z_i^\star, \tau^\star, T^\star$ 的微小偏移。

If any spacetime-filling D3-branes are present, the open string moduli z_{D3} that measure their positions inside X_6 appear in the Pfaffian $\mathcal{A}$, because 3-7 strings are charged under G and so affect the gaugino condensate:

如果存在充满时空的 D3 膜，用来度量它们在 X_6 内部位置的开弦模 z_{D3} 会出现在 Pfaffian $\mathcal{A}$ 中，因为 3-7 弦带有 G 的荷，因此会对金微子凝聚产生影响:

$$W = W_0 + \mathcal{A}(z_i, \tau, z_{D3})\, e^{-2\pi T/c(G)} + \dots . \tag{74}$$

At the level of equation counting, this dependence is sufficient to stabilize the D3- brane positions z_{D3} : the D3-brane moduli space would have been all of X_6 in the absence of the gaugino condensate superpotential, but the actual moduli space is a set of isolated points. This has been verified explicitly in toroidal orientifold compactifications and in certain local models [81]. It also accords with the general expectation that generic $\mathcal{N} = 1$ supergravity theories do not have exact moduli spaces.

从方程数目的角度来看，这种依赖关系足以稳定 D3 膜位置 z_{D3} : 在不存在金微子凝聚超势的情况下，D3 膜模空间本应是整个 X_6 ，但实际模空间是一组孤立点。这一点已经在环面定向紧化和某些局域模型中得到了显式验证 [81]。它也符合一般预期: 一般的 $\mathcal{N} = 1$ 超引力理论不存在精确模空间。

We made two crucial assumptions in the above discussion: we supposed that there exists a consistent choice of quantized flux such that W_0 is exponentially small, and we supposed that the divisor D supports a gaugino condensate on D7- branes. A natural question is whether there exist actual Calabi-Yau orientifolds, and explicit choices of quantized flux, which fulfill these conditions and give rise to a supersymmetric AdS_4 vacuum. This question has been resolved in the affirmative by direct construction [80, 82, 83] , as we will now explain.

我们在上述讨论中做了两个关键假设: 我们假设存在一个自洽的量子化通量选择，使得 W_0 是指数小量，且除子 D 可以支持 D7 膜上的金微子凝聚。一个自然的问题是: 是否存在真实的卡拉比-丘定向形，以及满足这些条件并产生超对称 AdS_4 真空的显式量子化通量选择？正如我们接下来会解释的，这个问题已经通过直接构造 [80, 82, 83] 得到了肯定的答案。

Mechanism for Small Flux Superpotential

小通量超势机制

The quantized fluxes F_3 and H_3 correspond to classes in $H^3(X_6, \mathbb{Z})$, which can be understood as a lattice of dimension $2h^{2,1}+2$. A specification of fluxes then corresponds to the choice of two lattice vectors $\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$. A choice of fluxes carries the D3-brane charge:

量子化通量 F_3 与 H_3 对应 $H^3(X_6, \mathbb{Z})$ 中的上同调类，可将其理解为一个维数为 $2h^{2,1}+2$ 的格。给定通量就相当于选择两个格向量 $\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$ 。所选通量携带 D3 膜电荷:

$$Q_{\text{flux}}^{\text{D3}} = \int_{X_6} H_3 \wedge F_3 = \frac{1}{2} \vec{f}^{\mathrm{T}} \cdot \sum \cdot \vec{h}. \tag{75}$$

Thus, not all $\vec{f}, \vec{h}$ are consistent: Gauss’ s law (13) limits fluxes to those with $Q_{\text{flux}}^{\text{D3}}$ no larger than the negative D3-brane charge of the orientifold planes. Flux vacua thus correspond to choices of lattice vectors in a bounded region. For large $h^{2,1}$, the number of allowed choices is vast.

因此，并非所有 $\vec{f}, \vec{h}$ 都是自洽的: 高斯定律 (13) 要求通量满足 $Q_{\text{flux}}^{\text{D3}}$ 不大于定向镜面的负 D3 膜电荷。因此通量真空对应有界区域内的格向量选择。当 $h^{2,1}$ 很大时，允许的选择数量极其庞大。

The KKLT scenario relies on the existence of fluxes $\vec{f}, \vec{h}$ such that the expectation value $|W_0|$ of the flux superpotential is exponentially small. Important early work on the statistics of flux vacua found the expected distribution of $|W_0|$ in the approximation that the fluxes are continuous [84]. The outcome was that the smallest expected value of $|W_0|$ could be exponentially small in $h^{2,1}$, but that at the same time such flux configurations were exponentially rare.

KKLT 方案依赖于满足通量超势能期待值 $|W_0|$ 为指数小量的通量 $\vec{f}, \vec{h}$ 的存在。早期关于通量真空统计的重要工作在通量连续近似下得到了 $|W_0|$ 的预期分布 [84]。结果表明，$|W_0|$ 的最小预期值可以是关于 $h^{2,1}$ 的指数小量，但同时这类通量构形是指数稀少的。

For another perspective,(31) shows that the problem of finding small $|W_0|$ is akin to finding a short vector in a high-dimensional lattice. The shortest vector problem is a well-known, cryptographically hard problem [85]. This is consistent with a picture in which finding exponentially small $|W_0|$ may be possible, but would require $h^{2,1} \gg 1$, and moreover would be exponentially costly [86, 87][20] .

换个角度看，式 (31) 表明寻找小 $|W_0|$ 的问题等价于在高维格中寻找短向量问题。最短向量问题是一个著名的密码学困难问题 [85]。这符合如下图景: 找到指数小的 $|W_0|$ 是可能的，但需要 $h^{2,1} \gg 1$ ，并且成本是指数级的 [86, 87][20] 。

This state of affairs appears discouraging, but there is important additional structure that can be exploited [83]. Length is measured in (31) by pairing with the period vector $\vec{\Pi}$. So suppose we can decompose:

这种现状看起来令人沮丧，但存在可以利用的重要额外结构 [83]。在式 (31) 中，长度是通过与周期向量 $\vec{\Pi}$ 做内积度量的。因此假设我们可以做如下分解:

$$\vec{\Pi} = \vec{\Pi}_{\text{poly}} + \vec{\Pi}_{\text{exp}}\,, \tag{76}$$

where $\vec{\Pi}_{\text{poly}}$ is polynomial and $\vec{\Pi}_{\text{exp}}$ is exponentially small, in a sense we will make precise. Then, the idea is to find configurations $\vec{f}, \vec{h}$ such that

其中 $\vec{\Pi}_{\text{poly}}$ 是多项式项，$\vec{\Pi}_{\text{exp}}$ 是指数小项，我们稍后会明确这一点。那么我们的思路就是找到满足下式的构形 $\vec{f}, \vec{h}$

$$\left(\vec{f} - \tau\vec{h}\right)^{\top} \cdot \sum \cdot \vec{\Pi}_{\text{poly}} \equiv 0, \tag{77}$$

i.e., flux choices that are automatically orthogonal to the ”big” part of $\vec{\Pi}$. In such a case, one will have:

即，通量选择自动正交于 $\vec{\Pi}$ 的“大”分量。在这种情况下，我们会得到:

$$|W_0| = \sqrt{\frac{2}{\pi}} \left| \left(\vec{f} - \tau\vec{h}\right)^{\top} \cdot \sum \cdot \vec{\Pi}_{\text{exp}} \right| \ll 1. \tag{78}$$

To realize this idea, one needs a decomposition of the form of (76), and a means of choosing fluxes that obey the orthogonality condition (77).

要实现这一思路，我们需要得到 (76) 形式的分解，以及找到满足正交条件 (77) 的通量选择方法。

Near the large complex structure (LCS) point in moduli space, the periods automatically obey (76). To see this, we recall that the prepotential $\mathcal{F}$ for the complex structure moduli z_i takes the form:

在模空间的大复结构 (LCS) 点附近，周期自动满足 (76) 式。我们回顾一下，复结构模 z_i 的预势 $\mathcal{F}$ 形式为:

$$\mathcal{F} = \mathcal{F}_{\text{poly}}\left(z_i\right) + \mathcal{F}_{\text{exp}}\left(z_i\right) = \mathcal{F}_{\text{poly}}\left(z_i\right) - \frac{1}{(2\pi i)^3} \sum_{\beta = c^i q_i} \text{GV}(\beta)\, \text{Li}_3\left(e^{2\pi i c^i z_i}\right).$$

(79)

Here $\widetilde{X}_6$ is the threefold that is mirror to X_6 ; the sum runs over curve classes $\beta \in H_2\left(\widetilde{X}_6, \mathbb{Z}\right)$, expressed in terms of coefficients c^i and a basis q_i; $\text{GV}(\beta)$ is the genus-0 Gopakumar-Vafa invariant of β; Li_3 is the trilogarithm; and $\mathcal{F}_{\text{poly}}\left(z_i\right)$ is a polynomial of degree three in the z_i . The sum of exponentials corresponds to the contributions of worldsheet instantons in compactification of type IIA string theory on $\widetilde{X}_6$.

此处 $\widetilde{X}_6$ 是 X_6 的镜像三维流形；求和遍历曲线类 $\beta \in H_2\left(\widetilde{X}_6, \mathbb{Z}\right)$ ，用系数 c^i 和基 q_i;$\text{GV}(\beta)$ 表示; β; Li_3 是零亏格 Gopakumar-Vafa 不变量；三对数是 $\mathcal{F}_{\text{poly}}\left(z_i\right)$ ；而 $\mathcal{F}_{\text{poly}}\left(z_i\right)$ 是 z_i 的三次多项式。指数求和项对应 IIA 型弦理论在 $\widetilde{X}_6$ 上紧化时世界面瞬子的贡献。

As usual in mirror symmetry, the practical approach is to compute $\mathcal{F}$ exactly in the duality frame where it is purely classical geometry: here, that means evaluating the periods of Ω on $\widetilde{X}_6$, which reduces to the problem of evaluating (rather elaborate) contour integrals, and can be automated. Thus, all the constants appearing in (79) are computable: the coefficients in $\mathcal{F}_{\text{poly}}$ are determined by the intersection numbers and Chern classes of $\widetilde{X}_6$, and the Gopakumar-Vafa invariants can be obtained from the mirror map [89].

和镜像对称中的通常做法一样，实用方法是在其对应纯经典几何的对偶框架中精确计算 $\mathcal{F}$: 在这里这意味着计算 Ω 在 $\widetilde{X}_6$ 上的周期，这可归为计算 (相当复杂的) 围道积分的问题，并且可以自动化完成。因此，(79) 中出现的所有常数都是可计算的: $\mathcal{F}_{\text{poly}}$ 中的系数由 $\widetilde{X}_6$ 的相交数和陈类确定，而戈帕库马尔-瓦法不变量可以从镜像映射得到 [89]。

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[20] For a counterargument, see [88].

[20] 关于相反的论证，参见 [88]。

On relating the periods $\vec{\Pi}$ to the prepotential, one easily shows that the decomposition (79) implies a decomposition (76), with $\vec{\Pi}_{\text{poly}}$ resulting from approximating $\mathcal{F} \approx \mathcal{F}_{\text{poly}}$, and $\vec{\Pi}_{\text{exp}}$ the correction resulting from the instanton series $\mathcal{F}_{\text{exp}}$.

在将周期 $\vec{\Pi}$ 与预备势联系起来时，很容易看出分解式 (79) 蕴含了分解式 (76)，其中 $\vec{\Pi}_{\text{poly}}$ 源于对 $\mathcal{F} \approx \mathcal{F}_{\text{poly}}$ 的近似，$\vec{\Pi}_{\text{exp}}$ 则是瞬子级数 $\mathcal{F}_{\text{exp}}$ 带来的修正。

The orthogonality condition (77) corresponds to a Diophantine equation in the flux integers. To see this, we change variables from $\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$ to $\mathbf{M}, \mathbf{K} \in \mathbb{Z}^{h^{2,1}}$ (see [80] for details) and define:

正交性条件 (77) 对应流量整数中的一个丢番图方程。为说明这一点，我们将变量从 $\vec{f}, \vec{h} \in \mathbb{Z}^{2h^{2,1}+2}$ 换为 $\mathbf{M}, \mathbf{K} \in \mathbb{Z}^{h^{2,1}}$ (详见文献 [80]) 并定义:

$$p^i := \left(\tilde{\kappa}_{ijk} M^k\right)^{-1} K_j, \tag{80}$$

where $\tilde{\kappa}_{ijk}$ are the triple intersection numbers of the mirror $\widetilde{X}_6$. Then, one finds that (77) holds for choices of flux obeying

其中 $\tilde{\kappa}_{ijk}$ 是镜像 $\widetilde{X}_6$ 的三重相交数。随后可以发现，(77) 对满足下式的流量选取成立:

$$\mathbf{p} \cdot \mathbf{K} = 0, \quad \text{with} \quad \mathbf{p} \in \mathcal{K}_{\widetilde{X}_6}, \tag{81}$$

where $\mathcal{K}_{\widetilde{X}_6}$ is the Kähler cone of $\widetilde{X}_6$. As always one must also impose the tadpole condition: we have $-\frac{1}{2}\mathbf{M} \cdot \mathbf{K} \equiv Q^{\text{D3}}_{\text{flux}}$, and so $\mathbf{M}, \mathbf{K}$ are constrained to obey

其中 $\mathcal{K}_{\widetilde{X}_6}$ 是 $\widetilde{X}_6$ 的凯勒锥。和往常一样，我们还必须施加蝌蚪条件: 我们有 $-\frac{1}{2}\mathbf{M} \cdot \mathbf{K} \equiv Q^{\text{D3}}_{\text{flux}}$，因此 $\mathbf{M}, \mathbf{K}$ 受约束满足:

$$\frac{1}{2}\mathbf{M}\cdot\mathbf{K} = Q_{\mathrm{loc}}^{\mathrm{D3}} \tag{82}$$

A choice of fluxes obeying (81) and (82), and thus obeying (77), is called a perturbatively flat vacuum, or PFV. In a PFV, one linear combination of the complex structure moduli and axiodilaton remains unfixed at the perturbative level (i.e., for $\mathcal{F} \approx \mathcal{F}_{\text{poly}}$), while the $h^{2,1}$ orthogonal directions are supersymmetrically stabilized by fluxes. We can choose to parameterize the perturbatively flat direction by τ . The effective superpotential for τ , which is dictated by the quantized fluxes and the Gopakumar-Vafa invariants, is a sum of exponentials:

满足 (81) 和 (82) 从而也满足 (77) 的流量选取被称为微扰平坦真空，简称 PFV。在微扰平坦真空中，复结构模和轴子膨胀子的一个线性组合在微扰层面 (即对 $\mathcal{F} \approx \mathcal{F}_{\text{poly}}$ 而言) 保持未固定，而正交的 $h^{2,1}$ 个方向由流量通过超对称方式定标。我们可以选择用 τ 参数化微扰平坦方向。由量子化流量和戈帕库马尔-瓦法不变量确定的 τ 的有效超势是指数项的和:

$$W_{\text{eff}}(\tau) = -\frac{1}{2^{3/2}\pi^{5/2}}\left(\sum_{\beta}\mathbf{M}\cdot\beta\,\mathrm{GV}(\beta)\,\mathrm{Li}_2\left(e^{2\pi i\tau\mathbf{p}\cdot\beta}\right)\right). \tag{83}$$

In some examples one finds a racetrack structure, where two terms compete in a region of weak coupling where all other terms are negligible [83].

在部分例子中会发现竞赛道结构，在弱耦合区域存在两项竞争，其他所有项都可忽略 [83]。

For example, there exists an O3/O7 orientifold with $h^{2,1}_{-} = 5$ and $h^{1,1}_{+} = 113$, and a choice of quantized fluxes therein, for which, in the normalizations of [80]:

例如，存在一个带有 $h^{2,1}_{-} = 5$ 和 $h^{1,1}_{+} = 113$ 的 O3/O7 orientifold，其中选取了一组量子化流量，在文献 [80] 的归一化下，有:

$$\sqrt{8\pi^5}W_{\text{eff}}(\tau) = -2e^{2\pi i\tau\cdot\frac{7}{29}} + 252e^{2\pi i\tau\cdot\frac{7}{28}} + \mathcal{O}\left(e^{2\pi i\tau\cdot\frac{43}{126}}\right). \tag{84}$$

The F-term for τ vanishes at $\tau = \tau^\star$ with $\mathrm{Im}\left(\tau^\star\right) \approx 0.011$. This example therefore has an exponentially small flux superpotential [80, 82]:

τ 的 F 项在 $\tau = \tau^\star$ 处为零，此时满足 $\mathrm{Im}\left(\tau^\star\right) \approx 0.011$ 。因此该例子存在一个指数小的流量超势 [80, 82]:

$$|W_0| = |W_{\text{eff}}(\tau_\star)| \approx \left(\frac{2}{252}\right)^{29} \approx 10^{-61}. \tag{85}$$

To recap, by finding fluxes that obey the Diophantine equation (77), one ensures that in the joint complex structure and axiodilaton moduli space, a single complex field remains massless at the perturbative level, while all other directions acquire large supersymmetric masses [21] . For the remaining PFV direction τ , the effective superpotential is a sum of exponentials, and so when a minimum exists at some $\tau^\star$, the vev of the superpotential $|W_0| = |W_{\text{eff}}(\tau_\star)|$ is naturally exponentially small. To engineer such a minimum, one can search through the discrete choices of topology - of X_6 and correspondingly its mirror $\widetilde{X}_6$ - and fluxes $\vec{f}, \vec{h}$ to find a case where W_{eff} is a racetrack, with two terms having nearly equal exponents and hierarchical prefactors. Examples of this sort have been found on a large scale: many billions of PFVs have been identified in Calabi-Yau threefolds [92].

总而言之，通过找到满足丢番图方程 (77) 的流量，我们可以保证: 在复结构轴子伸缩子联合模空间中，微扰水平下仅单个复场保持无质量，其余所有方向都会获得大的超对称质量 [21] 。对于剩余的 PFV 方向 τ ，有效超势是多个指数项的和，因此当某个 $\tau^\star$ 处存在极小值时，超势的真空期望值 $|W_0| = |W_{\rm eff}(\tau_\star)|$ 自然是指数小的。要构造这样的极小值，我们可以遍历 X_6 及其镜像 $\widetilde{X}_6$ 的离散拓扑选择与流量 $\vec{f}, \vec{h}$ ，找到满足 $W_{\rm eff}$ 为跑道型的情况，即两个指数项的指数几乎相等，prefactor 呈层级结构。这类例子已经被大规模发现: 人们已经在卡拉比-丘三维流形中识别出数十亿个 PFV [92]。

Small values of $|W_0|$ in PFVs are far more prevalent than continuous flux models predict, and occur at much smaller values of $h^{2,1}$, because the smallness of $|W_0|$ does not rely on searching in a high-dimensional lattice [22] . The example of (85) has $h^{2,1} = 5$, and it is the values of the Gopakumar-Vafa invariants and flux integers that dictate the exponential structure.

PFV 中 $|W_0|$ 取小值的情况远比连续流量模型的预测更普遍，并且可以在小得多的 $h^{2,1}$ 处出现，这是因为 $|W_0|$ 的小值并不依赖于在高维格点 [22] 中搜索。式 (85) 的例子就给出了 $h^{2,1} = 5$ ，正是戈帕库马尔-法瓦不变量和流量整数决定了指数结构。

In summary, exponentially small values of $|W_0|$ can be found systematically by constructing PFVs. This relies on computing the topology of mirror pairs $(X_6, \widetilde{X}_6)$ of Calabi-Yau threefolds; computing the periods; finding O3/O7 orientifold projections; and solving the Diophantine equation (77) in the flux integers. Searching in a high-dimensional lattice is not required, and so the PFV approach represents a dramatic reduction in complexity.

综上，通过构造 PFV 可以系统地得到指数小的 $|W_0|$ 。该过程依赖于: 计算卡拉比-丘三维流形镜像对 $(X_6, \widetilde{X}_6)$ 的拓扑；计算周期；确定 O3/O7 定向轨投影；求解流量整数满足的丢番图方程 (77)。该方法不需要在高维格点中搜索，因此 PFV 方法大幅降低了问题复杂度。

Non-perturbative Superpotential

非微扰超势

We now turn to computing the non-perturbative superpotential $W_{\rm np}$ for the Kähler moduli, as defined in (42). In section ”Non-perturbative Superpotential” we explained the conditions under which an effective divisor D supports a Euclidean D3-brane or gaugino condensate superpotential term. To stabilize all $h^{1,1}_+$ Kähler moduli T_a in a compactification, one needs to ensure that there are contributions to $W_{\rm np}$ that depend on each of the T_a [23] . Moreover, there are infinitely many candidate divisors, and to conclusively demonstrate the existence of a vacuum for the Kähler moduli, one must show that the terms that one computes and includes are in fact dominant over all omitted terms.

我们现在开始计算 (42) 式定义的凯勒模的非微扰超势 W_{np} 。我们曾在“非微扰超势”一节中解释了有效除子 D 支持欧几里得 D3 膜或戈奇诺凝聚超势项的条件。要在紧致化中稳定所有 $h_{+}^{1,1}$ 凯勒模 T_a ，必须确保 W_{np} 存在依赖于每个 T_a[23] 的贡献。此外，候选除子有无穷多个，要最终证明凯勒模真空存在，必须证明我们计算并引入的项对所有忽略项而言确实是占主导的。

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[21] For a variation on the PFV setup that allows further structure, see [90], while for a mechanism using asymptotic Hodge theory, see [91].

[21] 关于 PFV 框架的变体可引入更多结构，参见 [90]，而利用渐近霍奇理论的机制参见 [91]。

[22] The reason for the disparity compared to continuous flux predictions is that quantized fluxes are a set of measure zero in the space of continuous fluxes, and statements about distributions in the continuous ensemble only hold up to sets of measure zero [80]. A statistical comparison appears in [93].

[22] 与连续通量预测存在差异的原因在于，量子化通量在连续通量空间中是零测集，连续系综的分布结论仅在零测集之外成立 [80]，统计比较可参见 [93]。

[23] See [94] for an analysis of the prospects for Kähler moduli stabilization in ensembles of geometries.

[23] 关于几何系综中凯勒模稳定前景的分析参见 [94]。

To this end, suppose that X_6 is a Calabi-Yau orientifold, $\{D_a\}, a = 1, \ldots, h^{1,1}$ is a basis of $H_4(X_6, \mathbb{Z})$, and $\{D_A\}, A = 1, \ldots, N \geq h^{1,1}$ is a set of irreducible divisors that generate the semigroup of effective divisors, i.e., such that every effective divisor D on X_6 can be written as a nonnegative integer linear combination of the D_A . We write $D_A = c_A^a D_a$ with $c_A^a \in \mathbb{Z}$. We can specify a point in the Kähler moduli space of X_6 by the complexified volumes T_a of the D_a .

为此，假设 X_6 是一个卡拉比-丘定向轨形，$\{D_a\}, a = 1, \ldots, h^{1,1}$ 是 $H_4(X_6, \mathbb{Z})$ 的一组基，$\{D_A\}, A = 1, \ldots, N \geq h^{1,1}$ 是生成有效除子半群的不可约除子集合，即 X_6 上的任意有效除子 D 都可以写成 D_A 的非负整数线性组合。我们记 $D_A = c_A^a D_a$ 满足 $c_A^a \in \mathbb{Z}$。我们可以通过复化体积 T_a 对 D_a 来指定 X_6 凯勒模空间中的一个点。

The orientifold action determines which divisors support D7-brane gauge groups. In favorable classes of compactifications, each O7-plane coincides with exactly four D7-branes, and gives rise to the gauge algebra $\mathfrak{so}(8)$, whose dual Coxeter number is $c(\mathfrak{so}(8)) = 6$. When the associated divisor is rigid in the sense of (37), the low-energy theory is pure glue super-Yang-Mills and generates a gaugino condensate superpotential. Suppose that on X_6 , a set $D_1, \ldots, D_k \left(k < h^{1,1}\right)$ of divisors are rigid and support $\mathfrak{so}(8)$ gauge algebras, while the remainder of the semigroup generators $D_{k+1}, \ldots, D_N$ are either not wrapped by D7-branes or else support nonrigid D7- brane stacks (e.g., with $\dim H_{+}^{\bullet}(D, \mathcal{O}_D) = (1, 0, 1)$).

定向轨形作用决定了哪些除子支持 D7 膜规范群。在优良类紧致化中，每个 O7 平面对应恰好四个 D7 膜，生成规范代数 $\mathfrak{so}(8)$ ，其对偶考克斯特数为 $c(\mathfrak{so}(8)) = 6$ 。当相关除子满足 (37) 式定义的刚性条件时，低能理论为纯胶超级杨米尔斯理论，会产生戈奇诺凝聚超势。假设在 X_6 上，除子集合 $D_1, \ldots, D_k\left(k < h^{1,1}\right)$ 是刚性的且支持 $\mathfrak{so}(8)$ 规范代数，其余半群生成元 $D_{k+1}, \ldots, D_N$ 要么不被 D7 膜包裹，要么支持非刚性 D7 膜堆 (例如带有 $\dim H_+^{\bullet}(D, \mathcal{O}_D) = (1,0,1)$ 的情况)。

There very often exists a point $T_a^{\star}$ at which

通常存在这样一个点 $T_a^{\star}$ ，满足

$$\frac{1}{6}\operatorname{Re}(T_1) \approx \ldots \approx \frac{1}{6}\operatorname{Re}(T_k) \approx \operatorname{Re}(T_{k+1}) \approx \ldots \approx \operatorname{Re}(T_{h^{1,1}}) \ll \operatorname{Re}(T_{h^{1,1}+1}), \ldots, \operatorname{Re}(T_N).$$

(86)

We will call a point obeying (86) a KKLT point.

我们将满足 (86) 式的点称为 KKLT 点。

At a KKLT point $T_a = T_a^{\star}$, Euclidean D3-branes wrapping any of the D_A with $A \in \left\{k+1, \ldots, h^{1,1}\right\}$ are hierarchically more important than Euclidean D3-branes wrapping the D_A with $A > h^{1,1}$, or wrapping nontrivial linear combinations of the D_A . The task of evaluating the leading Euclidean D3-brane superpotential terms then reduces to the finite task of counting the fermion zero modes of the D_A for $A \in \left\{k+1, \ldots h^{1,1}\right\}$, i.e., of computing $\dim H_{\pm}^i\left(D_A, \mathcal{O}_{D_A}\right)$ for $i = 0,1,2$.

在 KKLT 点 $T_a = T_a^{\star}$ 处，欧几里得 D3 膜包裹任何满足 $A \in \left\{k+1, \ldots, h^{1,1}\right\}$ 的 D_A ，其效应层级上远大于欧几里得 D3 膜包裹满足 $A > h^{1,1}$ 的 D_A 、或包裹 D_A 的非平凡线性组合的效应。因此，计算领头阶欧几里得 D3 膜超势项的任务可简化为有限问题: 计数 $A \in \left\{k+1, \ldots h^{1,1}\right\}$ 对应的 D_A 的费米子零模，即对 $i = 0,1,2$ 计算 $\dim H_{\pm}^i\left(D_A, \mathcal{O}_{D_A}\right)$ 。

If all of $D_{k+1}, \ldots, D_{h^{1,1}}$ are rigid and not wrapped by D7-branes, the total superpotential takes the form:

如果所有 $D_{k+1}, \ldots, D_{h^{1,1}}$ 都是刚性的，且没有被 D7 膜包裹，总超势的形式为:

$$W = W_{\text{flux}} + \sum_{a=1}^{k} \mathcal{A}_a(z_i, \tau, z_{D3})\, e^{-2\pi T_a/6} + \sum_{a=k+1}^{h^{1,1}} \mathcal{A}_a(z_i, \tau, z_{D3})\, e^{-2\pi T_a} + \ldots$$

(87)

where the omitted terms are hierarchically smaller than the terms shown [24] .

其中省略项层级上小于已写出的项 [24] 。

[24] The terms in the Euclidean D(-1)-brane superpotential (43) are no larger than $e^{-\pi\tau}$, so if $g_s \ll 1$ and if W_{flux} arises from a PFV in which the dominant terms in (83) have $\mathbf{p}\cdot\beta < 1/2$, we can safely neglect $W_{\mathrm{ED}(-1)}$. Moreover, it was shown in [95] that $W_{\mathrm{ED}(-1)}$ vanishes identically in the case that all D7-brane gauge algebras are $\mathfrak{so}(8)$. See [80],[95] for discussions of this point.

[24] 欧几里得 D(-1) 膜超势 (43) 中的项不大于 $e^{-\pi\tau}$，因此若 $g_s \ll 1$，且 W_{flux} 来自 PFV，其中 (83) 的主导项满足 $\mathbf{p}\cdot\beta < 1/2$，我们就可以放心忽略 $W_{\text{ED}(-1)}$。此外，文献 [95] 已证明，当所有 D7 膜规范代数均为 $\mathfrak{so}(8)$ 时，$W_{\text{ED}(-1)}$ 恒为零。关于这一点的讨论参见 [80]、[95]。

We call (87) a KKLT superpotential. In the special case that all the divisors $D_1, \ldots, D_{h^{1,1}}$ are not just rigid, but also pure rigid - meaning that their uplifts to divisors of the F-theory fourfold have trivial intermediate Jacobians - and that the compactification contains no spacetime-filling D3-branes, the Pfaffian prefactors of the pure rigid superpotential terms are simply constants $\mathcal{A}_a$, and so we have

我们将 (87) 称为 KKLT 超势。在特殊情形下: 若所有除子 $D_1, \ldots, D_{h^{1,1}}$ 不仅是刚性的，还是纯刚性的——即它们提升到 F 理论四维体的除子后具有平凡中间雅可比，且紧致化中不充满时空的 D3 膜，则纯刚性超势项的 Pfaffian 前置因子仅为常数 $\mathcal{A}_a$，因此我们得到

$$W = W_{\text{flux}} + \sum_{a=1}^{k} \mathcal{A}_a e^{-2\pi T_a/6} + \sum_{a=k+1}^{h^{1,1}} \mathcal{A}_a e^{-2\pi T_a} + \ldots \tag{88}$$

The conditions leading to (88) may appear highly restrictive, but compactifications furnishing pure rigid KKLT superpotentials have been found at large $h^{1,1}$ [80, 92] .

得到 (88) 的条件看起来约束性很强，但在大 $h^{1,1}$ [80, 92] 下已经找到了能给出纯刚性 KKLT 超势的紧致化。

In a compactification with a KKLT point $T_a^\star$ (86) and a KKLT superpotential (87), the dominant terms in W_{np} are exactly those shown in (87). In such a case, the effective theory for the Kähler moduli T_a is well-approximated by (87) and (46), and the task of stabilizing the Kähler moduli is purely computational: one needs to check whether a supersymmetric vacuum exists in a regime of control, at some point T_a^{vac} very near $T_a^\star$.

在具有 KKLT 点 $T_a^\star$ (86) 和 KKLT 超势 (87) 的紧致化中，W_{np} 的主导项恰好就是 (87) 中给出的那些项。在这种情况下，Kähler 模 T_a 的有效理论可以用 (87) 和 (46) 很好地近似，稳定 Kähler 模的任务完全是计算性的: 只需要在可控制区域内，在非常靠近 $T_a^\star$ 的某点 T_a^{vac} 处，验证是否存在超对称真空。

Requirements for KKLT Vacua

KKLT 真空的要求

The KKLT scenario for de Sitter vacua requires a Calabi-Yau orientifold flux compactification that fulfills the following criteria:

KKLT 德西特真空场景要求卡拉比-丘定向模通量紧化满足以下判据:

a. The fluxes stabilize the complex structure moduli and axiodilaton at a point $(z^\star, \tau^\star)$ in moduli space where $|W_0|$ is exponentially small.

a. 通量将复结构模和轴子伸缩子稳定在模空间中 $|W_0|$ 呈指数小的点 $(z^\star, \tau^\star)$ 。

b. The total superpotential, including non-perturbative terms, takes the form [25] (87).

b. 包含非微扰项的总超势具有形式 [25] (87)。

c. The point $(z^\star, \tau^\star)$ is near a conifold singularity in the moduli space, and the fluxes threading the conifold give rise to a Klebanov-Strassler [96] throat region.

c. 点 $(z^\star, \tau^\star)$ 靠近模空间中的锥奇点，贯穿锥的通量会产生克莱巴诺夫-斯特拉斯勒 [96] 喉区。

d. The warp factor and geometry of the throat are such that the inclusion of one or more anti-D3-branes metastably breaks supersymmetry [97] and leads to a de Sitter vacuum.

d. 喉的翘曲因子与几何特性使得引入一个或多个反 D3 膜会亚稳破缺超对称 [97]，并得到德西特真空。

e. The resulting vacuum occurs in a parameter regime where all the approximations being made are valid: $g_s \ll 1$; the Kähler moduli are stabilized at a point $T^\star$ where the α' expansion is under control; and the backreaction effects of D-branes, fluxes, and orientifolds on the leading-order solution are controllably small.

e. 最终得到的真空处于所有所用近似都成立的参数区间: $g_s \ll 1$ ； 凯勒模被稳定在 α' 展开可控的点 $T^\star$ ； 且 D 膜、通量和定向模对领头阶解的反作用效应可控且很小。

Conditions (a),(b), and (e) together are sufficient for an $\mathcal{N} = 1$ supersymmetric AdS_4 vacuum, while (a),(b),(c), and (e) suffice for an $\mathcal{N} = 1$ supersymmetric AdS_4 vacuum with a conifold region. The further condition (d) is necessary for a de Sitter vacuum.

条件 (a)、(b) 和 (e) 一起足以得到一个 $\mathcal{N} = 1$ 超对称 AdS_4 真空，而 (a)、(b)、(c) 和 (e) 足以得到一个带锥区域的 $\mathcal{N} = 1$ 超对称 AdS_4 真空。额外的条件 (d) 是得到德西特真空的必要条件。

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[25] A form more general than (87), involving multiple distinct dual Coxeter numbers, would suffice for the KKLT mechanism, but because examples of (87) exist and are simpler to describe, we will restrict to them here.

[25] 涉及多个不同对偶考克斯特数、比 (87) 更一般的形式也适用于 KKLT 机制，但由于 (87) 的实例已经存在且描述更简单，我们在此仅讨论这种形式。

Explicit Constructions

显式构造

To illustrate the ideas explained above, we will now describe explicit examples of flux compactifications on orientifolds of Calabi-Yau threefolds, with all moduli stabilized, which are incarnations of the KKLT scenario. We will begin with the case of $\mathcal{N} = 1$ supersymmetric AdS_4 vacua without warped throats [80], i.e., vacua fulfilling (a), (b), and (e).

为阐明上述思路，我们接下来介绍卡拉比-丘三维形定向拟射影上通量紧致化的显式实例，这些实例是 KKLT 方案的具体实现，所有模都已稳定。我们先从没有翘曲喉的 $\mathcal{N} = 1$ 超对称 AdS_4 真空开始 [80]，即满足条件 (a)、(b)、(e) 的真空。

Suppose that X_6 is a Calabi-Yau threefold hypersurface in a toric variety obtained from the Kreuzer-Skarke list. Computational advances summarized in section ”Computational Advances” make it possible to compute intersection numbers [98], O3/O7 orientifold actions [24], the topology of divisors and their uplifts to F-theory [99,100], and the Gopakumar-Vafa invariants of curves [89] in X_6 and $\widetilde{X}_6$. Combined with a choice of quantized fluxes (29), these data suffice to determine the flux superpotential (31) and the non-perturbative superpotential terms from Euclidean D3-branes (37) and gaugino condensates (40), so that the full superpotential (41) is known.

假设 X_6 是 Kreuzer-Skarke 列表给出的 toric 簇中的卡拉比-丘三维形超曲面。“计算进展”一节总结的计算方法进步使得我们可以在 X_6 和 $\widetilde{X}_6$ 中计算相交数 [98]、O3/O7 定向拟射影作用 [24]、除子的拓扑及其提升到 F-理论的结果 [99,100]，以及曲线的 Gopakumar-Vafa 不变量 [89]。结合量子化通量的选择 (29)，这些数据足以确定通量超势 (31)，以及来自欧氏 D3 膜 (37) 和戈金诺凝聚 (40) 的非微扰超势项，从而得到完整超势 (41)。

In section ”Mechanism for Small Flux Superpotential” we described a mechanism for achieving (a): one chooses quantized fluxes obeying (81) and (82), leading to a PFV whose effective superpotential $W_{\text{eff}}\,(\tau)$ is a sum of exponentials (83), and selects a case in which the leading terms in (83) compete in a two-term racetrack [83], so that $|W_0| \ll 1$. One can accomplish this in practice for X_6 a Calabi-Yau threefold hypersurface with $h^{2,1} \lesssim 8$.

在“小通量超势的机制”一节中我们描述了实现条件 (a) 的机制: 选取满足 (81) 和 (82) 的量子化通量，得到的有效超势 $W_{\text{eff}}\,(\tau)$ 是指数项 (83) 的和，当 (83) 中的主导项在双项赛道中相互竞争 [83]，即可得到 $|W_0| \ll 1$ 。对于 X_6 这类具有 $h^{2,1} \lesssim 8$ 的卡拉比-丘三维形超曲面，我们可以实际完成这一构造。

In section ”Non-perturbative Superpotential” we summarized the zero mode computation that determines the non-perturbative superpotential for the Kähler moduli, and so allows one to check condition (b). Calabi-Yau threefold hypersur-faces with $h^{1,1} \gg 1$ tend to have many - often $h^{1,1}$ or $h^{1,1}+1$ - pure rigid divisors, in which case W takes the KKLT form (88). One can then check whether a KKLT point $T_a^{\star}$ (86) exists in a region of control [26] .

在“非微扰超势”一节中，我们总结了确定凯勒模非微扰超势的零模计算，借此可以检验条件 (b)。具有 $h^{1,1} \gg 1$ 的卡拉比-丘三维形超曲面通常存在大量纯刚性除子，数量常常达到 $h^{1,1}$ 甚至 $h^{1,1}+1$ ，此时 W 满足 KKLT 形式 (88)，我们进而可以检验 KKLT 点 $T_a^\star$ (86) 是否存在于可控区域 [26] 中。

Examples have been found that fulfill all these conditions, in threefolds with $4 \leq h^{2,1} \leq 7$ and $51 \leq h^{1,1} \leq 214$. In the example of (85), with $h^{2,1} = 5$ and $h^{1,1} = 113$, the string coupling is polynomially small, $g_s \approx 0.011$, and the vacuum energy is proportional to

已经在具有 $4 \leq h^{2,1} \leq 7$ 和 $51 \leq h^{1,1} \leq 214$ 的三维形中找到了满足所有这些条件的实例。在 (85) 的实例中，当 $h^{2,1} = 5$ 和 $h^{1,1} = 113$ 时，弦耦合是多项式小量 $g_s \approx 0.011$ ，真空能正比于

$$\left(\frac{2}{252}\right)^{58} \approx 10^{-122} \tag{89}$$

corresponding to an extraordinary degree of scale separation [80]. Extensive analysis of the control of corrections due to worldsheet instantons appears in [80], and rests on direct computation of Gopakumar-Vafa invariants to high degree [27] .

对应极高的标度分离度 [80]。[80] 中对世界面瞬子修正的可控性做了 extensive 分析，该分析建立在对直到高次 [27] 的 Gopakumar-Vafa 不变量直接计算的基础上。

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[26] Furthermore, one can systematically incorporate the leading corrections to the condition (73). These corrections, which result from the finite size of $1/|W_0|$, and also from the α'^3 and worldsheet instanton contributions to the Kähler potential and Kähler coordinates inherited from the parent $\mathcal{N} = 2$ compactification, can all be computed explicitly [80].

[26] 此外，我们可以系统地纳入条件 (73) 的主导修正。这些修正源于 $1/|W_0|$ 的有限尺寸，也源于源自母 $\mathcal{N} = 2$ 紧致化的凯勒势和凯勒坐标的 α'^3 和世界面瞬子贡献，所有这些修正都可以显式计算 [80]。

We now turn to examples with warped throat regions. The condition (c) was studied in [103, 104] , and was shown to be compatible with (a): one can extend the PFV mechanism of [83] to the vicinity of conifolds. Stabilization of the Kähler moduli, condition (b), was not addressed in [103, 104], though no obstacle to stabilization was identified.

我们现在来看带扭曲颈区域的例子。条件 (c) 已在 [103, 104] 中研究，结果表明它与 (a) 相容: 可以将 [83] 的 PFV 机制推广到锥邻域。凯勒模的稳定，即条件 (b)，并未在文献 [103, 104] 中讨论，不过也未发现存在稳定化障碍。

The full set of conditions, (a)-(e), are addressed in [92]: explicit examples have been found in which (a), (b), and (c) are fulfilled, and (d) and (e) hold to leading order in the α' expansion, i.e., omitting α' corrections [105, 106] to the potential of the anti-D3-brane in the warped throat. Thus, at leading order in α' , these configurations are metastable KKLT de Sitter vacua. We comment on related issues in section ”Anti-D3-Branes”.

全套条件 (a)-(e) 已在文献 [92] 中讨论: 已经找到了满足 (a)(b)(c) 的显式例子，且 (d) 和 (e) 在 α' 展开领头阶成立，即省略了扭颈中反 D3 膜势的 α' 修正 [105, 106] 。因此，在 α' 领头阶下，这些构造是亚稳的 KKLT 德西特真空。我们在“反 D3 膜”一节讨论相关问题。

Ten-Dimensional Description

十维描述

To understand how a KKLT vacuum can arise from the interplay of localized sources, fluxes, and the quantum effects of Euclidean D3-branes and gaugino condensates, it is instructive to obtain a ten-dimensional description of the solution, as originally proposed in [107], and to compare this with the EFT that has been determined from general studies in non-perturbative supersymmetric field theory [108-111]. A consistent ten-dimensional configuration should agree with the four-dimensional results.

为了理解 KKLT 真空如何由定域源、流量以及欧氏 D3 膜和戈迪诺凝聚的量子效应相互作用产生，按照文献 [107] 最初的思路，对该解进行十维描述，并将其与非微扰超对称场论通用研究中得到的 EFT[108-111] 对比，是很有启发意义的。自洽的十维构型应当与四维结果一致。

Gaugino condensation is a four-dimensional effect, but in string compacti-fications the gaugino bilinear expectation value $\langle\lambda\lambda\rangle$ can be computed in the four-dimensional theory and included as a source term localized on D7-branes. This requires knowing the couplings of D7-brane gauginos to bulk fields [112, 113], such as

戈迪诺凝聚是一种四维效应，但在弦紧致化中，戈迪诺双线性期待值 $\langle\lambda\lambda\rangle$ 可以在四维理论中计算，并将其作为定域在 D7 膜上的源项纳入。这需要知道 D7 膜戈迪诺对体场的耦合 [112,113]，例如

$$\mathcal{L} \supset c\int_D \sqrt{g}G_3 \cdot \Omega\overline{\lambda\lambda} \tag{90}$$

where c is a constant and D is the divisor wrapped by the D7-branes. Setting $\overline{\lambda\lambda} \to \langle\overline{\lambda\lambda}\rangle \neq 0$, the coupling (90) becomes a source term for G_3 that is localized on D .

其中 c 是常数，D 是 D7 膜缠绕的除子。取 $\overline{\lambda\lambda} \to \langle\overline{\lambda\lambda}\rangle \neq 0$ 后，耦合式 (90) 就成为定域在 D 上、对应 G_3 的源项。

One can then seek a solution of the ten-dimensional supergravity equations of motion with these localized sources. The appropriate framework is compactification on manifolds with dynamic SU(2) structure [114]: the gaugino condensate sources a localized deviation away from the conformally Calabi-Yau solution of the classical theory [107, 113, 115, 116] , including IASD flux profiles.

我们接下来可以在这些定域源的基础上, 寻找十维超引力运动方程的解。合适的框架是带动态 SU(2) 结构的流形紧致化 [114]: 戈迪诺凝聚会导致经典理论共形卡鲁扎-丘解 [107, 113, 115, 116] 产生定域偏离，包括 IASD 流量分布。

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[27] String loop corrections to the Kähler potential have not been computed explicitly in the examples of [80], but their effects are parametrically suppressed at weak coupling and large volume, and so will be negligible in the vacua of [80] unless there are accidental enhancements by many orders of magnitude. Advances in computing string loop corrections to the Kähler potential appear in [101, 102].

[27] 文献 [80] 的例子中并未显式计算凯勒势的弦圈修正，但在弱耦合和大体积下，这类修正的效应在参数层面是被压低的，因此除非出现意外的多个数量级增强， 否则在文献 [80] 的真空中可以忽略。凯勒势弦圈修正的相关研究进展可见文献 [101, 102]。

Consistency requires that the four-dimensional curvature $\mathcal{R}_4^{10\text{ d sol.}}$ obtained from the ten-dimensional equations of motion should match the curvature $\mathcal{R}_4^{\text{EFT}}$ obtained from the four-dimensional Einstein equations equipped with the KKLT scalar potential obtained from (21), (71). The undertaking of precisely matching the ten-dimensional and four-dimensional results by computing $\mathcal{R}_4^{10\text{ d sol.}}$ and comparing to $\mathcal{R}_4^{\text{EFT}}$ was initiated in [117] and pursued in [118-122]. Essential data for making this comparison comes from the ten-dimensional supersymmetry conditions (i.e., Killing spinor equations) that account for the gaugino condensate: these conditions were proposed in [107], building on [114], and further studied in [123].

自洽性要求，从十维运动方程得到的四维曲率 $\mathcal{R}_4^{10\text{ d sol.}}$ ，应当与由 (21)、(71) 得到 KKLT 标势后，通过四维爱因斯坦方程算出的四维曲率 $\mathcal{R}_4^{\text{EFT}}$ 一致。通过计算 $\mathcal{R}_4^{10\text{ d sol.}}$ 并与 $\mathcal{R}_4^{\text{EFT}}$ 对比来精确匹配十维和四维结果的工作始于文献 [117]，并在文献 [118-122] 中得到推进。进行该对比的核心信息来自考虑戈迪诺凝聚的十维超对称条件 (即基灵旋量方程): 这些条件是在文献 [114] 的基础上由文献 [107] 提出，并在文献 [123] 中得到进一步研究。

It was shown in [124] and [125] that the supersymmetry conditions of [107] do not provide a consistent representation of the gaugino condensate. The corrected Killing spinor equations necessary for consistency were obtained in [124] at leading order in $\langle\lambda\lambda\rangle$, and then in general in [125]. Equipped with the corrected supersymmetry conditions, [124] found an exact match:

文献 [124] 和 [125] 指出，文献 [107] 的超对称条件无法自洽描述戈迪诺凝聚。满足自洽性要求的修正基灵旋量方程最先由文献 [124] 在 $\langle\lambda\lambda\rangle$ 领头阶得到，随后文献 [125] 给出了一般形式。利用修正后的超对称条件，文献 [124] 得到了精确匹配:

$$\mathcal{R}_4^{10\text{ d sol.}} = \mathcal{R}_4^{\text{EFT}}, \tag{91}$$

which is strong evidence that the ten-dimensional solution is a consistent representation of the four-dimensional effective theory. The question of localized divergences in this setting remains open [125, 126].

这是证明十维解是四维有效理论自洽描述的有力证据。该框架下定域发散的问题目前仍未解决 [125, 126]。

LVS

大体积场景 (LVS)

We reviewed in section ”Tree Level” that the tree-level effective theory in type IIB flux compactifications has no-scale structure, cf. (55), and so the Kähler moduli T_a are unstabilized at tree level. Dependence of the scalar potential on the Kähler moduli arises from corrections to the tree-level data:

我们在“树图水平”一节已经回顾过，IIB 型通量紧致化的树图水平有效理论具有无标度结构，参见 (55)，因此凯勒模 T_a 在树图水平未被稳定。标量势对凯勒模的依赖来源于对树图水平数据的修正:

$$W = W_{\text{tree}} + \delta W,\ K = K_{\text{tree}} + \delta K. \tag{92}$$

Recalling the general form (20), we have $\delta W = W_{\text{np}}$ and $\delta K = K_{\text{pert.}} + K_{\text{np}}$, and as before we use W_0 to denote the expectation value of W_{tree} in the supersymmetric vacuum configuration of the complex structure moduli and axiodilaton.

回顾一般形式 (20)，我们得到 $\delta W = W_{\text{np}}$ 和 $\delta K = K_{\text{pert.}} + K_{\text{np}}$ ，和之前一样，我们用 W_0 表示复结构模与轴子 dilaton 超对称真空构型下 W_{tree} 的期望值。

Including the corrections δK and δW in the scalar potential will lift the original flatness of the potential:

标量势中包含修正项 δK 和 δW 后，原势能的平坦性会被解除:

$$V = V_{\delta K} + V_{\delta W} + \ldots \tag{93}$$

The tree-level contribution proportional to $|W_0|^2$ vanishes, thanks to the no-scale property (55), and the corrections due to δK and δW are of order:

得益于无标度性质 (55)，正比于 $|W_0|^2$ 的树图水平贡献为零，δK 和 δW 带来的修正阶数为:

$$V_{\delta K} \sim |W_0|^2 \delta K,\ V_{\delta W} \sim |W_0|\, \delta W + (\delta W)^2. \tag{94}$$

One would like to find regions of moduli space where the portions of $V_{\delta K}$ and $V_{\delta W}$ that are computable actually suffice to ensure a controlled minimum for the moduli potential [28] . The possible hierarchies that will be relevant are then [29]

我们希望找到模空间中的区域，其中 $V_{\delta K}$ 和 $V_{\delta W}$ 可计算的部分确实足以保证模势能 [28] 存在可控制的极小值。相关的可能等级关系为 [29]

$$|W_0| \sim \delta W \gg \delta K \tag{95}$$

$$|W_0| \gg \delta K \sim \delta W \tag{96}$$

$$|W_0| \gg \delta K \gg \delta W \tag{97}$$

We have already discussed (95): the KKLT scenario relies on choosing fluxes such that $\delta W \sim |W_0|$. In this case $V_{\delta K} \sim |W_0|^2 \delta K \ll V_{\delta W} \sim |W_0|^2$, and so we can neglect the corrections δK . The leading-order contribution to the scalar potential $V_{\delta W}$ can then give rise to a minimum, as we explained in section ”KKLT”.

我们已经讨论过 (95):KKLT 方案依赖于选择满足 $\delta W \sim |W_0|$ 的通量。在此情形下 $V_{\delta K} \sim |W_0|^2 \delta K \ll V_{\delta W} \sim |W_0|^2$ ，因此我们可以忽略修正项 δK 。正如我们在“KKLT”一节解释的，标量势 $V_{\delta W}$ 的领头阶贡献就能产生一个极小值。

However, because δK is polynomial in inverse volumes $1/T_a$, whereas $\delta W \equiv W_{\text{np}}$ is exponential in volumes, it is arguably more natural to suppose that $\delta K \gtrsim \delta W$. That is, although we showed explicitly in section ”Explicit Constructions” that one can find configurations obeying (95) and stabilizing all moduli, one might expect (96) and (97) to occur more often in the landscape of flux vacua.

但由于 δK 是逆体积 $1/T_a$ 的多项式，而 $\delta W \equiv W_{\text{np}}$ 是体积的指数函数，因此可以说假设 $\delta K \gtrsim \delta W$ 更为自然。也就是说，尽管我们在“显式构造”一节明确证明了可以找到满足 (95) 且稳定所有模的构型，我们依然可以预期 (96) 和 (97) 在通量真空景观中更常出现。

Stabilization achieved exclusively through perturbative corrections to K , as in (97), is an interesting possibility explored in [128]. For weak coupling and large volumes, the first nonvanishing term in the perturbative expansion will dominate, giving rise to a runaway potential and therefore no nontrivial minima at weak coupling. This is the celebrated Dine-Seiberg problem (see Fig. 1).

如 (97) 那样，仅通过对 K 的微扰修正实现的模稳定是文献 [128] 中研究的一个有趣方向。在弱耦合大体积下，微扰展开中第一个非零项会占主导，产生逃逸势，因此在弱耦合下不存在非平凡极小值。这就是著名的迪内-塞伯格问题 (参见图 1)。

To avoid this outcome in the regime (97), one would need to understand δK well enough to engineer a competition between two or more terms that suffices to stabilize all moduli in a region of control. This difficult task has not yet been achieved [30] .

为了在区间 (97) 中避免这个结果，我们需要充分理解 δK ，以构造出两个或多个项之间的竞争，从而在可控制区域稳定所有模。这项困难的任务目前尚未完成 [30] 。

We now turn to the more fruitful case (96), in which δK and δW are in competition. The main point is that the generic Calabi-Yau manifold has many

现在我们转向更有研究价值的情形 (96)，其中 δK 和 δW 相互竞争。核心点在于，一般卡拉比-丘流形存在许多

[28] Computing K_{np} systematically is currently out of reach, so we will only study regions where $\delta K \approx K_{\text{pert.}}$, and we therefore omit K_{np} henceforth.

[28] 系统地计算 K_{np} 目前超出了我们的能力范围，因此我们仅研究满足 $\delta K \approx K_{\text{pert.}}$ 的区域，此后我们将省略 K_{np} 。

[29] The cases (95) and (96) could be seen not as two alternative scenarios, but instead as different regimes for exploring the same class of models, in the sense that a scan of values of the flux superpotential can smoothly interpolate from one class to the other, as was shown in the example of $\mathbb{CP}^4[1,1,1,6,9]$ in [127].

[29] 情形 (95) 和 (96) 不应被看作两种不同的方案，而应被看作同一类模型的不同研究区间，这是因为正如文献 [127] 中 $\mathbb{CP}^4[1,1,1,6,9]$ 的例子所展示的，对通量超势的取值扫描可以实现两类区间之间的平滑过渡。

[30] An interesting perturbative approach to moduli stabilization was outlined recently in [78]. There is a concrete case in QFT in which a perturbation expansion can be re-summed: as is well-known, the renormalization group provides logarithmic corrections to the EFT. For string moduli these are re-summations in an expansion in $\alpha \log \mathcal{V}$, with α a weak coupling parameter. Following the Dine-Seiberg logic, this structure would naturally lead to a minimum in which the expansion parameter is $\mathcal{O}(1)$, which would imply $\mathcal{V} \sim e^{1/\alpha} \gg 1$. This would be an alternative way to fix the volume modulus consistent with weak coupling, and would naturally lead to exponentially large volumes. It would be very interesting to achieve explicit string realizations of this scenario.

[30] 近期在文献 [78] 中提出了一种有趣的模稳定微扰方法。在量子场论中存在一个可重求和微扰展开的具体情形: 众所周知，重整化群会给有效场论带来对数修正。对弦模而言，这些就是对 $\alpha \log \mathcal{V}$ 展开的重求和，其中 α 是弱耦合参数。遵循 Dine-Seiberg 的逻辑，该结构会自然得到一个极小值，其中展开参数为 $\mathcal{O}(1)$ ，这意味着 $\mathcal{V} \sim e^{1/\alpha} \gg 1$ 。这是固定体积模的另一种方法，与弱耦合自洽，且自然得到指数级大的体积。若能给出这个场景具体的弦实现，会非常有价值。

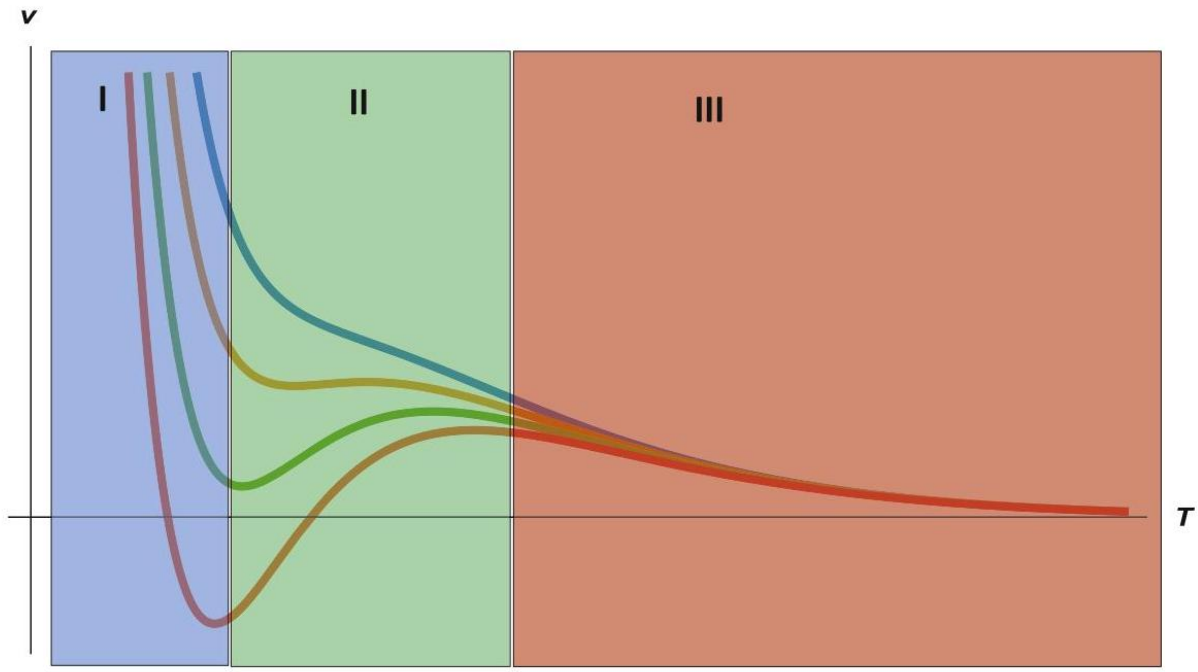

Fig. 1 The Dine-Seiberg problem. For large volumes and weak coupling, the scalar potential runs away toward the infinite volume or zero coupling limit. Region III is the region in which calculations in the EFT can be arbitrarily well controlled. The natural place to find a minimum, if one exists, is in region I, since it is here that quantum effects could most readily counterbalance the leading-order runaway behavior. Obtaining a minimum in the physically interesting region II, with finitely large volume and weak coupling, is more challenging and requires extra sources of hierarchies. Examples of such hierarchies are those achieved by a choice of fluxes in the KKLT scenario, or by competition of perturbative and non-perturbative terms in LVS

> 图 1 Dine-Seiberg 问题。大体积、弱耦合下，标量势会朝着无穷体积或零耦合极限跑走。区域 III 是有效场论计算可以任意好地控制的区域。如果存在极小值，最自然出现在区域 I，因为在这里量子效应最容易抵消领头阶的跑走行为。要在物理上有意义的区域 II(体积有限大且耦合弱) 得到极小值难度更高，需要额外的等级来源。这类等级的例子包括 KKLT 场景中通过选择流量得到的等级，或是大体积场景中微扰项与非微扰项相互竞争产生的等级

Kähler moduli. One combination of the moduli gives the volume $\mathcal{V}$ that controls the overall α' expansion. However, as we have seen, each of the Kähler moduli may be an expansion parameter: Euclidean D3-branes on a rigid four-cycle D with $\mathrm{Vol}(D) = c^a\tau_a$ give $\delta W \sim e^{-2\pi c^a \tau_a}$. If $W_0 = \mathcal{O}(1)$, we have $V_{\delta K} \sim \delta K$ and $V_{\delta W} \sim \delta W$. So if $\delta K \sim 1/\mathcal{V}$ and $\delta W \sim e^{-2\pi c^a \tau_a}$, the scalar potential may develop a minimum where $\delta K \sim \delta W$, i.e., at

> 凯勒模。模的一个组合给出体积 $\mathcal{V}$ ，控制整体 α' 展开。但正如我们所见，每个凯勒模都可以作为展开参数: 欧几里得 D3 膜包裹刚性四周期 D , 当 $\mathrm{Vol}(D) = c^a\tau_a$ 时给出 $\delta W \sim e^{-2\pi c^a \tau_a}$ 。若 $W_0 = \mathcal{O}(1)$ ，我们有 $V_{\delta K} \sim \delta K$ 和 $V_{\delta W} \sim \delta W$ 。因此如果 $\delta K \sim 1/\mathcal{V}$ 且 $\delta W \sim e^{-2\pi c^a \tau_a}$ ，标量势可以形成一个极小值，其中 $\delta K \sim \delta W$ ，也就是在

$$V \sim e^{2\pi c^a \tau_a}, \tag{98}$$

and for $\tau_a \gg 1$, the volume $\mathcal{V}$ will be exponentially large in the minimum. This is the origin of the name Large Volume Scenario (LVS) [129,130].

且当 $\tau_a \gg 1$ 时, 极小值处的体积 $\mathcal{V}$ 会是指数级大的。这就是大体积场景 (LVS) 名称的起源 [129,130]。

Let us now be more explicit. Following our discussion of perturbative corrections to the Kähler potential in section "Perturbative Corrections", we consider the leading-order α' correction at string tree level: this is the α'^3 correction of [66], cf. (63):

现在我们给出更具体的推导。结合“微扰修正”一节中对凯勒势微扰修正的讨论，我们考虑弦树图阶的领头阶 α' 修正: 这就是文献 [66] 中的 α'^3 修正，参见式 (63):

$$K_{\alpha'^3} = -\ln\left(-i\int_{X_6} \Omega\left(z_i\right) \wedge \overline{\Omega}\left(\bar{z}_i\right)\right) - \ln\left(-i\left(\tau - \overline{\tau}\right)\right) - 2\ln\left(\mathcal{V} + \frac{\xi}{2}\left(\frac{\tau - \overline{\tau}}{2i}\right)^{3/2}\right),$$

(99)where

其中

$$\xi = -\frac{\zeta\left(3\right)\chi\left(X_6\right)}{2(2\pi)^3}. \tag{100}$$

Here $\chi\left(X_6\right)$ is the Euler number of the corresponding Calabi-Yau manifold and ζ is the Riemann ζ function. $\mathcal{N} = 1$ corrections to the Kähler potential, yet to be computed, are expected to modify this coefficient.

此处 $\chi\left(X_6\right)$ 是对应卡拉比-丘流形的欧拉数，ζ 是黎曼 $\zeta\,\zeta$ 函数。尚未计算完成的 $\mathcal{N} = 1$ 对凯勒势的修正预计会改变这个系数。

The simple modification (99) of the Kähler potential changes the structure of the scalar potential, once it is combined with the flux plus non-perturbative superpotential (41):

凯勒势的简单修正 (99) 会在和流量加非微扰超势 (41) 结合后，改变标量势的结构:

$$W = W_0 + \sum_{a=1}^{n} \mathcal{A}_a e^{-2\pi c^a T_a} \tag{101}$$

where the index a runs over all effective divisors, with the understanding that $\mathcal{A}_a$ may vanish for some of them if the corresponding cycle is not rigid. Thus, n may be smaller than, equal to, or larger than $h^{1,1}$, depending on the threefold in question.

其中指标 a 遍历所有有效除子，约定如果对应周期不是刚性的，$\mathcal{A}_a$ 对部分除子可以为零。因此 n 可以小于、等于或大于 $h^{1,1}$ ，具体取决于所考虑的三 fold。

The leading-order scalar potential is positive-definite in the dilaton and complex structure moduli directions, and therefore to leading order, these fields are stabilized by the fluxes, as in KKLT, at $D_{z_i}W = 0$ and $D_\tau W = 0$.

领头阶标量势在 dilaton(胀子) 和复结构模方向是正定的，因此正如 KKLT 中的情况，领头阶下这些场会被流量稳定在 $D_{z_i}W = 0$ 和 $D_\tau W = 0$ 处。

The remaining potential as a function of the Kähler moduli reads:

作为凯勒模函数的剩余势可写为:

$$V = e^K \Big[K_{T_a} K^{T_a \bar{T}_b} W \bar{W}_{\bar{T}_b} + W_{\ T_a} K^{T_a \bar{T}_b} \left(\bar{W}_{\overline{T_b}} + K_{\overline{T_b}} \bar{W} \right)$$

$$+ \left(K_{T_a} K^{T_a \bar{T}_b} K_{\overline{T_b}} - 3 \right) |W|^2 \Big] . \tag{102}$$

Inserting the expressions (101) and (99) in (102), the scalar potential can be written as a sum of three terms (quadratic, linear, and independent of W_0 , respectively) [131]:

将式 (101) 和 (99) 代入 (102)，标量势可拆分为三项之和 (分别是二次项、一次项和独立于 W_0 的项)[131]:

$$V = V_{\alpha'^3} + V_{\mathrm{np1}} + V_{\mathrm{np2}}, \tag{103}$$

where

其中

$$V_{\alpha'^3} = e^K |W_0|^2 \times \frac{3\hat{\xi}\left(\mathcal{V}^2 + 7\mathcal{V}\hat{\xi} + \hat{\xi}^2\right)}{\left(\mathcal{V} - \hat{\xi}\right)\left(2\mathcal{V} + \hat{\xi}\right)^2}, \tag{104}$$

$$V_{\mathrm{np1}} = e^K |W_0| \times \sum_a 2 |\mathcal{A}_a| e^{-c_a \tau_a} \cos\left(b_a \theta_a - \phi_a + \theta_W\right)$$

$$\times \left[\frac{\left(4\mathcal{V}^2 + \hat{\xi}\mathcal{V} + 4\hat{\xi}^2\right)}{\left(\mathcal{V} - \hat{\xi}\right)\left(2\mathcal{V} + \hat{\xi}\right)} c_a \tau_a + \frac{3\hat{\xi}\left(\mathcal{V}^2 + 7\hat{\xi}\mathcal{V} + \hat{\xi}^2\right)}{\left(\mathcal{V} - \hat{\xi}\right)\left(2\mathcal{V} + \hat{\xi}\right)^2} \right], \tag{105}$$

$$V_{\mathrm{np2}} = e^K \sum_{a,b} |\mathcal{A}_a| \, |\mathcal{A}_b| \, e^{-(c_a \tau_a + c_b \tau_b)} \cos\left(c_a \theta_a - c_b \theta_b - \phi_a + \phi_b\right)$$

$$\times \Big[-\left(4\mathcal{V} + 2\hat{\xi}\right) \left(\kappa_{abc} t^c\right) c_a c_b + \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}} \left(c_a \tau_a\right) \left(c_b \tau_b\right)$$

$$+ \frac{4\mathcal{V}^2 + \hat{\xi}\mathcal{V} + 4\hat{\xi}^2}{\left(\mathcal{V} - \hat{\xi}\right)\left(2\mathcal{V} + \hat{\xi}\right)} \left(c_a \tau_a + c_b \tau_b\right) + \frac{3\hat{\xi}\left(\mathcal{V}^2 + 7\hat{\xi}\mathcal{V} + \hat{\xi}^2\right)}{\left(\mathcal{V} - \hat{\xi}\right)\left(2\mathcal{V} + \hat{\xi}\right)^2} \Big]. \tag{106}$$

Here $\hat{\xi} := \xi / g_s^{3/2}$, and ϕ_a and θ_W are the phases of $\mathcal{A}_a$ and W_0 , respectively.

此处 $\hat{\xi} := \xi / g_s^{3/2}$ 、ϕ_a 和 θ_W 分别是 $\mathcal{A}_a$ 和 W_0 的相位。

The scalar potential (103) is singular at $\mathcal{V} = \hat{\xi}$ for $\hat{\xi} > 0$ and $\mathcal{V} = -\hat{\xi}/2$ for $\hat{\xi} < 0$, but in any case validity of the perturbative expansion requires $\mathcal{V} \gg \hat{\xi}$, so these singularities will not be relevant. In general the domain of validity of the scalar potential is also limited by the fact that the Kähler moduli need to lie in the Kähler cone.

标量势 (103) 在 $\mathcal{V} = \hat{\xi}$ 的 $\hat{\xi} > 0$ 处和 $\mathcal{V} = -\hat{\xi}/2$ 的 $\hat{\xi} < 0$ 处奇异，但微扰展开的适用条件要求 $\mathcal{V} \gg \hat{\xi}$ ，因此这些奇点不影响讨论。一般来说，标量势的适用域还受限于凯勒模必须处于凯勒锥内这一条件。

The first term (104) vanishes as $\xi \to 0$, as it should in order to recover the tree-level result in this case. For $\mathcal{V} \gg \hat{\xi}$, we see from (99) that e^K scales as $1/\mathcal{V}^2$, so the first term (104) scales as $1/\mathcal{V}^3$, whereas (105) and (106) scale as $1/\mathcal{V}^2$ and $1/\mathcal{V}$, respectively, but also depend on powers of $e^{-\tau_a}$, making the moduli stabilization calculation very rich. An important technical point is that V_{np2} depends explicitly on the two-cycle moduli t^a , and in general it is not possible to determine $t^a(\tau_b)$ analytically. In practical terms it is therefore easier to consider the scalar potential as a function of the t^a fields.

第一项 (104) 按 $\xi \to 0$ 趋于零，这正是恢复树级结果所需要的。对于 $\mathcal{V} \gg \hat{\xi}$ ，我们从 (99) 中看到 e^K 标度为 $1/\mathcal{V}^2$ ，因此第一项 (104) 标度为 $1/\mathcal{V}^3$ ，而 (105) 和 (106) 分别标度为 $1/\mathcal{V}^2$ 和 $1/\mathcal{V}$ ，同时二者还依赖于 $e^{-\tau_a}$ 的幂次，这让模稳定的计算内容非常丰富。一个重要的技术要点是 V_{np2} 显式依赖于二周期模 t^a ，一般无法解析求解 $t^a(\tau_b)$ 。因此实际操作中，将标量势视为 t^a 场的函数处理会更简单。

The formula (104) is very general and holds for any number of moduli [31] . Let us consider key illustrative cases:

式 (104) 非常通用，适用于任意数量的模 [31] 。我们来看几个关键的示例情况:

- $h^{1,1} = 1$. This is the simplest case considered by KKLT. Here there is only one Kähler modulus, T_1 , and one triple intersection number $\kappa_{111} := k$. We assume that there is a non-perturbative contribution, i.e., we take $n = h^{1,1} = 1$, and

- $h^{1,1} = 1$ 。这是 KKLT 研究的最简单情形。此时只有一个凯勒模 T_1 ,和一个三重交点数 $\kappa_{111} := k$ 。我们假设存在非微扰贡献，即取 $n = h^{1,1} = 1$ ，且

$$\mathcal{V} = \frac{1}{6}kt^3,\ \tau_1 = \frac{1}{2}kt_1^2,\ \mathcal{V} = \frac{1}{3}\sqrt{\frac{2}{k}}\tau_1^{3/2},\ W = W_0 + \mathcal{A}e^{-cT_1}.$$

(107)

Since it is possible to express $\mathcal{V}$ as a function of τ_1 , we can work with the variable τ_1 . Equation (103) reproduces the KKLT potential for $\hat{\xi} = 0$:

由于可以将 $\mathcal{V}$ 表示为 τ_1 的函数，我们可以直接用变量 τ_1 来计算。对 $\hat{\xi} = 0$ ，式 (103) 重现了 KKLT 势:

$$V = \frac{9akg_s e^{K_{cs}}}{\tau_1^2}\,|\mathcal{A}|\,e^{-c\tau_1}\left[|W_0|\cos(c\theta_1 + \theta_W - \phi_1) + \frac{|\mathcal{A}|}{3}e^{-c\tau_1}(c\tau_1 + 3)\right]$$

(108)which reproduces the supersymmetric AdS minimum of KKLT. Nonvanishing values of $\hat{\xi}$ add corrections to the KKLT potential that have been considered in [133] and that can contribute to uplifting the original AdS minimum. Since there is only one Kähler modulus, there is no LVS minimum.

它重现了 KKLT 的超对称反德西特 (AdS) 极小值。$\hat{\xi}$ 的非零值会给 KKLT 势带来修正，这些修正已在文献 [133] 中讨论过，可用于抬升原本的 AdS 极小值。由于只有一个凯勒模，因此不存在大体积场景 (LVS) 极小值。

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[31] Here we have neglected the orientifold-odd moduli. A more general expression including these moduli can be found in [132].

[31] 此处我们忽略了 orientifold 奇模。包含这些模的更一般表达式可在文献 [132] 中找到。

- $h^{1,1} = 2$. This is the simplest case allowing an LVS minimum. Depending on the nature of the divisors and the triple intersection numbers, the volume may or may not be expressed explicitly in terms of the variables τ_1 and τ_2 . Also in this case, we may or may not have non-perturbative dependence of W on both moduli $(n = 1, 2)$. If $n = 2$ this allows a generalization of KKLT for two moduli. If $n = 1$ the second modulus needs to be stabilized by perturbative effects.

- $h^{1,1} = 2$。这是允许存在 LVS 极小值的最简单情况。根据除子的性质和三重相交数，体积可以或不能用变量 τ_1 和 τ_2 显式表示。在这种情况下，W 对两个模 $(n = 1, 2)$ 也可能存在或不存在非微扰依赖。若 $n = 2$，这就可以将 KKLT 推广到双模情况。若 $n = 1$，第二个模需要通过微扰效应来稳定。

The simplest example has X_6 an orientifold of the Calabi-Yau threefold hypersurface in weighted projective space $\mathbb{CP}^4[1, 1, 1, 6, 9]$. In this case,

最简单的例子是 X_6 加权射影空间 $\mathbb{CP}^4[1, 1, 1, 6, 9]$ 中卡拉比-丘三维超曲面的定向模。在这种情况下:

$$\tau_1 = \frac{t_1^2}{2}, \ \tau_2 = \frac{(t_1 + 6t_2)^2}{2}, \ 6\mathcal{V} = 3t_1^2 t_2 + 18t_1 t_2^2 + 36t_2^3 = \tau_2^{3/2} - \tau_1^{3/2}.$$

(109)

The scalar potential for $\mathcal{V} \gg \hat{\xi}$ takes the form:

$\mathcal{V} \gg \hat{\xi}$ 的标量势形式为:

$$V \simeq g_s e^{K_{cs}} \left[\frac{\alpha}{\mathcal{V}^3} + \frac{\beta \tau_2 e^{-c_1 \tau_1} \cos(c_1 \theta_1 + \theta_W - \phi_1)}{\mathcal{V}^2} + \frac{\gamma \sqrt{\tau_1} e^{-2c_1 \tau_1}}{\mathcal{V}} \right]$$

(110)

where

其中

$$\alpha = \frac{3\hat{\xi}|W_0|^2}{4},\ \beta = 2\,|\mathcal{A}|\,|W_0|\,c_1,\ \gamma = \frac{4}{3}c_1^2|\mathcal{A}|^2. \tag{111}$$

Extremizing with respect to θ_1 sets the cosine term to -1, and then extremizing with respect to τ_1 and the volume $\mathcal{V}$ (or τ_2) gives rise to exponentially large volume if $\alpha > 0$ (i.e., if the Euler characteristic $\chi(X_6)$ is negative):

对 θ_1 取极值可将余弦项置为-1，随后对 τ_1 和体积 $\mathcal{V}$ (或 τ_2) 取极值，若满足 $\alpha > 0$ (即欧拉示性数 $\chi(X_6)$ 为负)，就会得到指数级大体积:

$$\langle\mathcal{V}\rangle \simeq \left\langle\tau_2^{3/2}\right\rangle \simeq e^{c_1\tau_1},\ \langle\tau_1\rangle \simeq \frac{\xi^{2/3}}{g_s} \gg 1. \tag{112}$$

Here we considered only the case $n = 1$, without non-perturbative superpotential dependence on τ_2 . This is justified for the hypersurface in $\mathbb{CP}^4\,[1,1,1,6,9]$ since the divisor with volume τ_2 is not rigid. The presence of fluxes may modify this conclusion and give rise to non-perturbative contributions (see for instance [134]). In any case, such terms are highly suppressed compared to the $1/\mathcal{V}$ dependence and can be neglected.

此处我们仅考虑了 $n = 1$ 的情况，非微扰超势不依赖于 τ_2 。这对于 $\mathbb{CP}^4\,[1,1,1,6,9]$ 中的超曲面是成立的，因为体积为 τ_2 的除子不是刚性的。通量的存在可能会改变这一结论，产生非微扰贡献 (例如见文献 [134])。无论如何，这类项与对 $1/\mathcal{V}$ 的依赖相比都被高度压低，可以忽略。

The above vacuum is the prototypical example of LVS. In this example, it was essential that the volume took the form:

上述真空是 LVS 的典型范例。在这个例子中，体积取以下形式是关键:

$$\mathcal{V} \propto \tau_2^{3/2} - \tau_1^{3/2} \tag{113}$$

so that $\mathcal{V}$ decreases as τ_1 increases. Since the cycle of volume τ_1 behaves like a hole in X_6 , volumes $\mathcal{V}$ with the structure (113) are said to be of Swiss cheese form. The generalization to more than two Kähler moduli is

因此 $\mathcal{V}$ 随 τ_1 增大而减小。由于体积为 τ_1 的闭链在 X_6 中表现得如同一个孔洞，具有结构 (113) 的体积 $\mathcal{V}$ 被称为瑞士奶酪形式。推广到两个以上凯勒模的形式为

$$\mathcal{V} \propto F(\tau_{\tilde{a}}) - G(\tau_{\hat{a}}) \tag{114}$$

where we have split the moduli τ_a into two classes, $\tau_{\tilde{a}}$ and $\tau_{\hat{a}}$, and F and G are positive homogeneous functions of degree 3/2 of their arguments, such that the volume $\mathcal{V}$ decreases with increasing $\tau_{\hat{a}}$. In the case (113), the corresponding divisor corresponds to a del Pezzo surface that can be shrunk to zero size without affecting the positivity of the volume.

其中我们将模 τ_a 分为两类，$\tau_{\bar{a}}$ 和 $\tau_{\hat{a}}$ ，F 和 G 是关于其自变量的 3/2 次正齐次函数，满足体积 $\mathcal{V}$ 随 $\tau_{\hat{a}}$ 增大而减小。在 (113) 的情况中，对应的除子是德尔佩佐曲面，它可以收缩到零尺寸而不影响体积的正性。

When G is given by

当 G 由下式给出

$$G = \sum_{\hat{a}} \tau_{\hat{a}}^{3/2} \tag{115}$$

then the shrinking divisors are called diagonal del Pezzo surfaces. Although the form (115) appears to be quite special, it is common at $h^{1,1} = 2$, appearing in 22 out of 39 Calabi-Yau threefolds from the Kreuzer-Skarke list at $h^{1,1} = 2$[131][32] . Furthermore, this structure can easily be generalized to the $h^{1,1} > 2$ cases.

那么这些可收缩除子被称为对角德尔佩佐曲面。尽管形式 (115) 看起来非常特殊，但它在 $h^{1,1} = 2$ 中十分常见: 在 Kreuzer-Skarke 列表的 $h^{1,1} = 2$[131][32] 处的 39 个卡拉比-丘三维流形中，有 22 个都存在该结构。此外，该结构可以很容易地推广到 $h^{1,1} > 2$ 的情况。

In the models with $h^{1,1} = 2$ for which the Swiss cheese structure is not present, equation (103) remains applicable. A systematic analysis of all the models with $h^{1,1} = 2$ in the Kreuzer-Skarke list shows that seven additional models may give rise to LVS minima even though they lack diagonal del Pezzo divisors. In these cases it is more convenient to extremize the scalar potential with respect to the two-cycle moduli t^a . The remaining ten models with $h^{1,1} = 2$ correspond to K3 or T^4 fibrations over $\mathbb{P}_1$. In these cases the volume modulus is linear in one of the t^a moduli and can be written as $\mathcal{V} \sim \tau_1\sqrt{\tau_2}$. These cases do not give rise to LVS vacua.

在不存在瑞士奶酪结构的含 $h^{1,1} = 2$ 模型中，方程 (103) 仍然适用。对 Kreuzer-Skarke 列表中所有含 $h^{1,1} = 2$ 模型的系统分析表明，即使缺少对角 del Pezzo 除子，仍有另外七个模型可能产生 LVS 极小值。在这些情况下，相对于双周期模 t^a 对标量势取极值会更方便。其余十个含 $h^{1,1} = 2$ 的模型对应 K3 或 T^4 纤维化于 $\mathbb{P}_1$ 之上。这类情况中体积模是其中一个 t^a 模的线性函数，可写为 $\mathcal{V} \sim \tau_1\sqrt{\tau_2}$ 。这些模型不会产生 LVS 真空。

- $h^{1,1} = 3$. Again formula (103) may be used to identify vacua in which all the moduli are stabilized. There are again extensions of KKLT and LVS vacua. Out of 305 different Calabi-Yau orientifolds, more than 120 allow for LVS minima, by having either a Swiss cheese structure $\mathcal{V} = \tau_3^{3/2} - \tau_2^{3/2} - \tau_1^{3/2}$ or a slightly modified form $\mathcal{V} = \tau_3^{3/2} - (\alpha\tau_2 + \beta\tau_1)^{3/2} - \tau_1^{3/2}$.

- $h^{1,1} = 3$ 。同样可以利用公式 (103) 识别所有模都被稳定的真空。这里同样存在 KKLT 真空和 LVS 真空的推广。在 305 个不同的卡拉比-丘定向轨形中，超过 120 个允许存在 LVS 极小值，这些模型要么具有瑞士奶酪结构 $\mathcal{V} = \tau_3^{3/2} - \tau_2^{3/2} - \tau_1^{3/2}$ ，要么具有稍微修改的形式 $\mathcal{V} = \tau_3^{3/2} - (\alpha\tau_2 + \beta\tau_1)^{3/2} - \tau_1^{3/2}$ 。

The case of $h^{1,1} = 3$ illustrates much richer structure than the previous cases. A substantial fraction of models cannot be minimized in terms of the τ_a , and one needs to minimize (104) with respect to the t^a .

Even though analytic results are difficult to extract, numerical analysis is enough to identify minima, several of them of the LVS type. This illustrates that there are more LVS minima than naively expected.

> $h^{1,1}=3$ 的情况展现出比之前情况丰富得多的结构。很大一部分模型无法通过 τ_a 得到极小值，需要相对于 t^a 对 (104) 极小化。虽然很难得到解析结果，但数值分析足以识别极小值，其中多个极小值属于 LVS 类型。这说明 LVS 极小值的数量比朴素预期更多。

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[32] Only 27 of the 39 examples are topologically inequivalent, according to the systematic counting of inequivalent threefolds in the Kreuzer-Skarke list in [135, 136].

> [32] 根据 [135,136] 中对 Kreuzer-Skarke 列表内不等价三维流形的系统计数，39 个例子中仅有 27 个是拓扑不等价的。

Some of these models have one rigid divisor, such as a del Pezzo surface, while the third modulus is a K3 fibration modulus that is not fixed at this order of perturbation theory, and whose fixing relies on higher-order corrections (such as those discussed in section "Perturbative Corrections") that scale with higher powers of $1/\mathcal{V}$, for example, as $\delta V \sim \mathcal{O}\left(\mathcal{V}^{-10/3}\right)$. These fibrations allow for interesting anisotropic compactifications [137] in which two of the dimensions are much larger than the other four. The flatness of the fiber moduli potentials has also been used as one of the leading models for string moduli inflation [138]: see section "Inflation in String Theory".

> 其中部分模型包含一个刚性除子，例如 del Pezzo 曲面，而第三个模是 K3 纤维化模，在该阶微扰论中未被固定，其稳定依赖于高阶修正 (如“微扰修正”一节讨论的修正)，这类修正随 $1/\mathcal{V}$ 的更高次幂标度，例如标度为 $\delta V \sim \mathcal{O}\left(\mathcal{V}^{-10/3}\right)$。这些纤维化允许存在有趣的各向异性紧致化 [137]，其中两维度比其余四维大得多。纤维模势的平坦性也使其成为弦模暴涨的主流模型之一 [138]: 参见“弦论中的暴涨”一节。

- $h^{1,1}\geq 4$. Cases with larger numbers of Kähler moduli have been less systematically explored, mostly due to computational limitations (however, see section "Computational Advances"), but they are all captured by formula (103). In cases where there are $n<h^{1,1}$ rigid divisors, the existence of an isolated vacuum relies on understanding higher-order perturbative corrections to the Kähler potential.

> - $h^{1,1}\geq 4$ 。由于计算能力的限制，凯勒模数量更多的情况尚未得到系统研究 (不过参见“计算进展”一节)，但公式 (103) 都可以涵盖这些情况。当存在 $n<h^{1,1}$ 个刚性除子时，孤立真空的存在依赖于对凯勒势的高阶微扰修正的理解。

The structures that appeared at small $h^{1,1}$, such as the appearance of del Pezzo surfaces and fiber moduli, continue at larger $h^{1,1}$: in fact, it is known that most Calabi-Yau manifolds are fibrations [139-142]. LVS minima are currently limited to $h^{1,1}<h^{2,1}$, which sets a bound on the exploration of these vacua.

> 小 $h^{1,1}$ 下出现的结构，例如 del Pezzo 曲面和纤维模的出现，在大 $h^{1,1}$ 下依然存在: 事实上，已知大多数卡拉比-丘流形都是纤维化 [139-142]。目前 LVS 极小值仅存在于 $h^{1,1}<h^{2,1}$ 中，这给这类真空的研究设置了限制。

Supersymmetry Breaking and de Sitter Uplift

超对称破缺与德西特提升

The vacua with stabilized moduli that we have described so far are AdS_4 solutions: these are $\mathcal{N} = 1$ supersymmetric in the case of KKLT, and non-supersymmetric in the case of LVS. The uplift paradigm [79] proposes that from an AdS_4 solution with $V = V_{\mathrm{AdS}} < 0$, one can incorporate a source of supersymmetry breaking such that

到目前为止我们介绍的模稳定真空都是 AdS_4 解:KKLT 情形中这些解是 $\mathcal{N} = 1$ 超对称的，LVS 情形中则是非超对称的。提升范式 [79] 提出，从一个具有 $V = V_{\mathrm{AdS}} < 0$ 的 AdS_4 解出发，我们可以引入一个超对称破缺源，使得

$$V \to V_{\mathrm{tot}} = V_{\mathrm{AdS}} + V_{\mathrm{up}} \tag{116}$$

has a metastable de Sitter minimum.

得到一个亚稳态德西特极小值。

Let us illustrate the idea in the simple example of a KKLT model with $h_+^{1,1} = 1$, as in section ”KKLT”. In the supersymmetric AdS_4 vacuum, the F-term potential (21) is

我们来用一个包含 $h_+^{1,1} = 1$ 的 KKLT 模型的简单例子说明这个思路，就像“KKLT”一节那样。在超对称 AdS_4 真空中，F 项势 (21) 为

$$V_{\mathrm{AdS}} = V_F = -3e^{K_{\mathrm{tree}}} |W|^2 + \dots , \tag{117}$$

with W given by (71), T obeying (73), and the omitted terms representing higher corrections that can be neglected for $|W_0| \ll 1$.

其中 W 由式 (71) 给出，T 满足式 (73)，省略项代表对 $|W_0| \ll 1$ 可以忽略的高阶修正。

The first idea for an uplift, due to Kachru, Pearson, and Verlinde (KPV) [97], proposes that p anti-D3-branes at the bottom of a Klebanov-Strassler warped throat region can break supersymmetry, with [33]

提升的第一个想法由 Kachru、Pearson 和 Verlinde(KPV)[97] 提出，认为位于克莱巴诺夫-斯特拉斯勒翘曲 throat 区域底部的 p 反 D3 膜可以破缺超对称，满足 [33]

$$V_{\mathrm{up}} = \frac{\mathcal{D} T_{\mathrm{D3}}}{\left(T + \bar{T}\right)^2} > 0, \tag{118}$$

[33] The denominator was given as $\left(T + \bar{T}\right)^3$ in [79] and corrected to $\left(T + \bar{T}\right)^2$, which properly accounts for the warping, in [143]: see the treatment in [144].

[33] 在文献 [79] 中分母给出的形式是 $(T+\bar{T})^3$，文献 [143] 将其修正为 $(T+\bar{T})^2$，修正后的形式正确考虑了翘曲效应: 参见文献 [144] 中的处理。

---

where

其中

$$\mathcal{D} = 2pe^{4A(y_{\mathrm{IR}})}. \tag{119}$$

Here T_{D3} is the D3-brane tension, y_{IR} is the location of the tip of the warped throat inside X_6 , and $e^{A(y)}$ is the warp factor from (22), evaluated in the Klebanov-Strassler solution [96]. The exponential smallness of the warp factor ensures that the supersymmetry breaking effect is small in string units, i.e., $\mathcal{D} \ll 1$.

这里 T_{D3} 是 D3 膜张力，y_{IR} 是翘曲 throat 尖端在 X_6 内部的位置，$e^{A(y)}$ 是式 (22) 中的翘曲因子，在克莱巴诺夫-斯特拉斯勒解 [96] 中求值。翘曲因子的指数小性质保证了超对称破缺效应在弦单位下很小，即 $\mathcal{D} \ll 1$ 。

For suitable values of $\mathcal{D}$, the total potential for T

对于 $\mathcal{D}$ 的合适取值，T 的总势

$$V_{\mathrm{KKLT}} := V_{\mathrm{AdS}} + V_{\mathrm{up}}, \tag{120}$$

with V_{AdS} given by (117) and V_{up} given by (118), has a metastable de Sitter minimum at a point $T = T^{\mathrm{dS}}$. With $T^{\star}$ the minimum (73) obtained from (117) alone, one finds that

V_{AdS} 由式 (117) 给出，V_{up} 由式 (118) 给出，在点 $T = T^{\mathrm{dS}}$ 处存在一个亚稳态德西特极小值。对于仅由式 (117) 得到的极小值 (73)，即对应 $T^{\star}$ ，可以得到

$$T^{\mathrm{dS}} - T^{\star} = \mathcal{O}\left(1/T^{\star}\right) = \mathcal{O}\left(1/\log\left(|W_0|^{-1}\right)\right) \ll 1, \tag{121}$$

so the Kähler modulus shifts by a small amount toward larger volume.

因此凯勒模向更大体积方向移动了一个小量。

We remark that the expression (120) arose from asserting that the F-term potential V_{AdS} and the anti-D3-brane energy V_{up} combine by simple addition. This is easily argued to be a good approximation in the four-dimensional EFT [79], and the ten-dimensional computations summarized in section ”Ten-Dimensional Description” provide independent evidence from a very different perspective [124].

我们注意到式 (120) 源于 F 项势 V_{AdS} 与反 D3 膜能量 V_{up} 简单相加的假设。在四维有效场论中，不难论证这是一个很好的近似 [79]，而“十维描述”一节总结的十维计算从完全不同的角度给出了独立证据 [124]。

The expressions (118) and (119) for the anti-D3-brane potential were obtained at leading order in α' by KPV [97]. At this level of approximation, explicit flux compactifications furnishing candidates for KKLT de Sitter vacua have been obtained in [92] (cf. [145]). These vacua are built on the PFV mechanism reviewed in section ”Mechanism for Small Flux Superpotential” and the resulting framework for supersymmetric AdS_4 vacua described in section ”Explicit Constructions”. However, computations of α' corrections to the KPV potential [105,106] indicate that in order to realize metastable de Sitter vacua that survive α' corrections, the curvature of the throat region must fulfill a condition that is more stringent than that assumed in KPV and achieved in [92].

反 D3 膜势的表达式 (118) 和 (119) 是 KPV[97] 在 α' 领头阶得到的。在这个近似下，文献 [92] 已经得到了能作为 KKLT 德西特真空候选的显式通量紧致化 (参见 [145])。这些真空建立在“小通量超势机制”一节回顾的 PFV 机制，以及“显式构造”一节描述的由此得到的超对称 AdS_4 真空框架之上。然而，对 KPV 势 α' 修正的计算 [105,106] 表明，要实现能保留 α' 修正的亚稳态德西特真空，throat 区域的曲率必须满足一个条件，该条件比 KPV 假设的、文献 [92] 中实现的条件更严格。

Anti-D3-Branes

反 D3 膜

Before we describe other ideas for sources of supersymmetry-breaking uplifts (in section ”Other Uplifting Mechanisms”), we will summarize the status of anti-D3- brane supersymmetry breaking.

在我们介绍其他超对称破缺 uplifting 源的方案之前 (在“其他 uplifting 机制”小节)，我们先总结反 D3 膜超对称破缺的研究现状。

The analysis of KPV relied on treating the anti-D3-brane as a probe of the Klebanov-Strassler geometry, and subsequent work pursued a backreacted solution [146, 147]. The consistency of the KPV proposal was questioned in [148], on the basis that the linearized solution produced by anti-D3-branes smeared on the S^3 has singularities resulting from three-form fluxes. Dymarsky obtained an $SU(2) \times SU(2)$ -invariant linearized solution with appropriate boundary conditions [149] (see also [150, 151]), showed that the ADM mass and the Coulomb potential for a D3-brane matched the probe computations [143], and further argued that the singularities in the fluxes were artifacts of linearization. Singularities and polarization were studied in, e.g., [144, 152, 153], and the effective field theory description of antibrane sources was used in [154, 155] to argue that instabilities are absent. It was then shown in [156, 157] (see also [158]) that the singularities were a consequence of the ansatz being too restrictive, and not allowing the polarized brane. The blackfold formalism was applied in [159] to study the anti-D3-brane in a broader region of parameter space, including finite temperature, and the results gave strong evidence for consistency of the metastable KPV state (see also [160-162]).

KPV 的分析将反 D3 膜视为克莱巴诺夫-斯特拉斯勒几何的探针，后续工作则得到了反作用解 [146, 147]。KPV 方案的自洽性曾在文献 [148] 中被质疑，其依据是弥散在 S^3 上的反 D3 膜得到的线性化解存在三形式通量导致的奇点。迪马尔斯基得到了满足适当边界条件的 $SU(2)\times SU(2)$ 不变线性化解 [149](另见 [150, 151])，证明 D3 膜的 ADM 质量和库仑势与探针计算结果一致 [143]，并进一步指出通量中的奇点是线性化带来的人为产物。奇点和极化已在文献 [144, 152, 153] 等中被研究，反膜源的有效场论描述被用在文献 [154, 155] 中论证不存在不稳定性。随后文献 [156, 157](另见 [158]) 指出，奇点是假设过于受限、不允许极化膜存在导致的结果。黑洞膜形式被应用在文献 [159] 中，用于研究更广泛参数空间 (包括有限温度) 下的反 D3 膜，结果为亚稳态 KPV 态的自洽性提供了强有力的证据 (另见 [160-162])。

A further issue arises in KKLT models as a result of the fine-tuning needed to achieve a de Sitter uplift [120, 163]. For the anti-D3-brane energy (118) to fall in the range such that (120) has a metastable minimum with positive vacuum energy, one needs the warp factor in (119) to obey

KKLT 模型中还存在另一个问题: 实现德西特 uplift 需要精细调节 [120, 163]。要让反 D3 膜能量 (118) 落在合适范围内，使得 (120) 存在具有正真空能的亚稳极小点，就要求 (119) 中的翘曲因子满足

$$e^{4A(y_{\mathrm{IR}})} \sim |W_0|^2. \tag{122}$$

The warp factor can be expressed in terms of the D3-brane charge $N_{\mathrm{D3}}^{\mathrm{throat}}$ and the curvature radius R_{throat} at the tip of the throat as [17]

翘曲因子可以用 throat 尖端的 D3 膜荷 $N_{\mathrm{D3}}^{\mathrm{throat}}$ 和曲率半径 R_{throat} 表示为 [17]

$$e^{4A(y_{\mathrm{IR}})} = \exp\left(-\frac{8\pi}{3}\frac{N_{\mathrm{D3}}^{\mathrm{throat}}}{R_{\mathrm{throat}}^4}\right), \tag{123}$$

while at the KKLT point $T_a^\star$, the Kähler moduli and W_0 are related by (73), i.e.,

而在 KKLT 点 $T_a^\star$，凯勒模与 W_0 满足关系式 (73)，即

$$\mathrm{Re}\, T_a^\star = \frac{c_a}{2\pi}\log\left(|W_0|^{-1}\right), \tag{124}$$

with c_a the dual Coxeter number for the gauge group on D_i. Because $c_a \in \{1, 6\}$ in all the examples of section "Explicit Constructions", and 6 is not a large number for the considerations at hand, we will set $c_a \to 1$ to simplify the discussion. Combining the relations (122),(123), and (124), and requiring $R_{\mathrm{throat}} \gtrsim 1$ so that the supergravity description of the throat is valid, we find:

其中 c_a 是 D_i 上规范群的对偶考克斯特数。由于在“显式构造”小节的所有例子中都满足 $c_a \in \{1, 6\}$，且对当前讨论而言 6 不是一个大数，我们将令 $c_a \to 1$ 来简化讨论。结合关系式 (122)、(123) 和 (124)，并要求 $R_{\mathrm{throat}} \gtrsim 1$ 以保证 throat 的超引力描述有效，我们得到:

$$N_{\mathrm{D3}}^{\mathrm{throat}} \gtrsim \mathrm{Re}\, T_a^\star. \tag{125}$$

The observation made in [120] and further investigated in [163] is that for some compactification geometries X_6 ,(125) implies that the warp factor e^A becomes negative in the bulk of X_6 , rather than just becoming negative in an exponentially small neighborhood of the O7-planes, where supergravity is in any case not the appropriate description. This issue is called the singular bulk problem [163].

文献 [120] 中给出、并在文献 [163] 中进一步研究的结论是: 对某些紧致化几何 X_6 ，(125) 意味着翘曲因子 e^A 在 X_6 的本体区域变为负数，而非仅在 O7 平面的指数小邻域内变负——无论如何超引力在 O7 平面附近本来就不是合适的描述。这个问题被称为本体奇点问题 [163]。

Whether the singular bulk problem is present in a given X_6 depends on how the overall volume $\mathcal{V}$ and the four-cycle volumes are related. Roughly speaking, $\mathcal{V}$ controls the size of the throat that can fit into X_6 without causing warp factor singularities to appear (see [120, 163, 164] for a precise statement). Taking the volumes to be related by

给定 X_6 是否存在本体奇点问题，取决于整体体积 $\mathcal{V}$ 与四周期体积之间的关系。粗略来说，$\mathcal{V}$ 控制了能容纳在 X_6 中、又不产生翘曲因子奇点的 throat 的大小 (精确表述见 [120, 163, 164])。假设体积满足关系

$$\mathcal{V} = \mathcal{P}\left(\mathrm{Re}\, T_a^\star\right)^{3/2} \tag{126}$$

with $\mathcal{P}$ a constant, one can check that if $\mathcal{P}$ is of order unity, the singular bulk problem is severe [120, 163].

其中 $\mathcal{P}$ 为常数，不难验证: 如果 $\mathcal{P}$ 是一阶单位量，本体奇点问题会非常严重 [120, 163]。

Taking $\mathcal{P} \sim 1$ is a reasonable first guess for a highly isotropic Calabi-Yau threefold, but given an explicit compactification, one can simply compute $\mathcal{P}$ at the KKLT point $T_a^\star$. In the example given in (89) of section ”Explicit Constructions”, one finds $\mathcal{P} \approx 8000$ [80]. The large value of $\mathcal{P}$ is a geometric fact about four-cycle sizes at KKLT points in threefolds with many moduli - cf. (198) - that very significantly ameliorates the singular bulk problem. This mechanism persists in de Sitter configurations: in the vacua of [92], in which the fine-tuning (122) is explicitly engineered and a de Sitter uplift occurs at leading order in α' , the singular bulk problem is again alleviated by geometric factors.

对于高度各向同性的卡拉比-丘三维流形，取 $\mathcal{P} \sim 1$ 是一个合理的初步猜测，但给定具体紧化后，我们可以直接在 KKLT 点 $T_a^\star$ 处计算 $\mathcal{P}$ 。在“具体构造”章节的式 (89) 给出的例子中，可得 $\mathcal{P} \approx 8000$ [80]。$\mathcal{P}$ 的较大取值是含多模三维流形 KKLT 点处四周期大小的几何特征——参见式 (198)——这能极大改善大体积奇异问题。该机制在德西特构型中仍然成立: 在文献 [92] 的真空里，该真空明确构造了微调式 (122)，并在 α' 的领头阶实现了德西特上抬，奇异体问题同样被几何因子缓解。

Nonlinear Supersymmetry and Nilpotent Superfields

非线性超对称与幂零超场

In order to study anti-D3-brane supersymmetry breaking, it is sometimes useful to repackage the supersymmetry-breaking effects in terms of a nilpotent chiral superfield X (see for instance [165-168]):

为了研究反 D3 膜超对称破缺，将超对称破缺效应重新整理为幂零手征超场 X 的形式有时会更为方便 (例如参见文献 [165-168]):

$$X = X_0 + \sqrt{2}\psi\theta + F\theta\bar{\theta},\ X^2 = 0 \Rightarrow X_0 = \frac{\psi\psi}{2F}. \tag{127}$$

The only propagating component of X is the fermion ψ , which is identified with the goldstino of nonlinearly realized supersymmetry. The nilpotent constraint substantially limits the possible X dependence of the superpotential and Kähler potential to be

X 唯一的传播分量是费米子 ψ ，它对应非线性实现超对称的戈德斯蒂诺。幂零约束极大限制了超势和凯勒势对 X 的依赖形式，得到:

$$W = W_0 + \rho X,\ K = K_0 + K_1 X + \bar{K}_1 \bar{X} + K_2 X\bar{X}, \tag{128}$$

with W_0, ρ, K_i functions of the other fields. In the case of a single Kähler modulus $T, K_0 = -3\log\left(T+\bar{T}\right)$, so in an expansion in $1/\left(T+\bar{T}\right)$, we can write [34] $K_2 = \left(T+\bar{T}\right)^{-\lambda}$, so that the scalar potential takes the general form:

其中 W_0, ρ, K_i 是其他场的函数。对于单个凯勒模 $T, K_0 = -3\log\left(T+\bar{T}\right)$ 的情况，我们可以对 $1/\left(T+\bar{T}\right)$ 做展开，写出 [34] $K_2 = \left(T+\bar{T}\right)^{-\lambda}$ ，由此标量势可以表示为如下一般形式:

$$V = V_0 + V_X,\ V_X = K^{-1}_{X\bar{X}}\left|\frac{\partial W}{\partial X}\right|^2 = \frac{|\rho|^2}{\left(T+\bar{T}\right)^{3-\lambda}}, \tag{129}$$

with V_0 the KKLT or LVS scalar potential. Notice that modular weight $\lambda = 1$ corresponds to a Kähler potential of the no-scale type, $K = -3\log\left(T+\bar{T}-X\bar{X}\right)$, and also for $\lambda = 1$, we have $V_X = V_{\mathrm{up}}$. This implies that the nilpotent superfield formalism naturally captures the effects of an anti-D3-brane. In sum, the Volkov-Akulov formalism of nonlinearly realized supersymmetry provides a convenient representation of anti-D3-brane supersymmetry breaking.

其中 V_0 是 KKLT 或 LVS 标量势。注意模权 $\lambda = 1$ 对应无标度型凯勒势 $K = -3\log\left(T+\bar{T}-X\bar{X}\right)$ ，且即便对于 $\lambda = 1$ ，我们也有 $V_X = V_{\mathrm{up}}$ 。这说明幂零超场形式论自然包含了反 D3 膜的效应。综上，非线性实现超对称的沃尔科夫-阿库洛夫形式论为反 D3 膜超对称破缺提供了一个便捷的描述。

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[34] This can be also argued in terms of the $SL(2,\mathbb{R})$ modular symmetry $T \to (aT - ib)/\ (icT + d)$, if X transforms as a modular form of weight λ .

[34] 这一点也可以通过 $SL(2,\mathbb{R})$ 模对称性 $T \to (aT - ib)/\ (icT + d)$ 来论证，只要 X 按权为 λ 的模形式变换。

Explicit orientifold compactifications have been found in which the spectrum of the anti-D3-brane at the tip of a throat consists of only the goldstino field, with the nilpotent superfield formalism describing the goldstino EFT couplings [168-170]: see also [171-174].

目前已经找到明确的定向紧化案例，其中位于喉部顶端的反 D3 膜谱仅包含戈德斯蒂诺场，幂零超场形式论可以描述戈德斯蒂诺的有效理论耦合 [168-170]: 另见文献 [171-174]。

Other Uplifting Mechanisms

其他上扬机制

Anti-D3-brane uplifting has been the best-studied mechanism to break supersym-metry in KKLT and to uplift the supersymmetric vacuum to a de Sitter vacuum. However, many other proposals exist, of which we will highlight a few [35]

反 D3 膜上扬一直是 KKLT 中破缺超对称、将超对称真空上扬为德西特真空研究最充分的机制。但仍存在许多其他方案，我们将在这里重点介绍其中几个 [35]

- Uplift from α' corrections. The α'^3 corrections to the Kähler potential given in (99) contribute:

- α' 修正带来的上扬。(99) 中给出的凯勒势能 α'^3 修正贡献如下:

$$V_{\alpha'^3} = \frac{\xi |W_0|^2}{\mathcal{V}^3}, \ \xi \propto -\chi(X_6)\,. \tag{130}$$

For $\xi > 0$ (which, in light of mirror symmetry, holds in half of all Calabi-Yau manifolds, setting aside cases with $\chi = 0$), this term contributes positively to the KKLT potential and may lift the minimum toward de Sitter. The main issue is control of the approximation: the term (130) needs to compete with lower-order terms present in the KKLT potential. For LVS the term (130) is responsible for the original non-supersymmetric AdS minimum. But scanning over different values of the flux superpotential, from the very small values required for KKLT to order $\mathcal{O}(1-100)$ as appears in the original LVS, other minima appear in different regimes, some of them with positive vacuum energy. However, although the volume is relatively large, it is usually not large enough for the approximations to be trusted [127]. For concrete examples of this mechanism, see for instance [133, 134, 176, 177].

对于 $\xi > 0$ (考虑镜像对称性，该关系在一半的卡拉比-丘流形成立，暂不讨论带有 $\chi = 0$ 的情况)，该项对 KKLT 势能做出正贡献，可将极小值向德西特方向抬升。核心问题在于近似的可控性:(130) 的项需要和 KKLT 势能中已有的低阶项竞争。对于大体积 scenarios(LVS)，(130) 的项就是最初非超对称反德西特极小值的来源。但对标量流超势的不同取值 (从 KKLT 要求的极小值到原始 LVS 中出现的 $\mathcal{O}(1-100)$ 量级)，不同区域会出现其他极小值，其中部分极小值真空能为正。然而，尽管流形体积相对较大，其大小通常不足以保证近似可靠 [127]。该机制的具体例子可见文献 [133, 134, 176, 177]。

- D-term uplift. The scalar potential for $\mathcal{N} = 1$ supersymmetric theories has two contributions: the F-term potential (21) that we have been considering so far and the D-term potential resulting from integrating out the D-term of a gauge superfield $\hat{V}^{\alpha}$:

- D 项上扬。$\mathcal{N} = 1$ 超对称理论的标量势能有两个贡献来源: 我们目前一直在讨论的 F 项势能 (21)，以及积分掉规范超场 $\hat{V}^{\alpha}$ 的 D 项后得到的 D 项势能:

$$V = V_F + V_D,\ V_D = \frac{1}{2} D^{\alpha} D_{\alpha}. \tag{131}$$

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[35] An alternative to the uplifts discussed here was proposed in [175]: quantum effects from the Standard Model sector may uplift an AdS vacuum with sufficiently small vacuum energy.

[35] 文献 [175] 提出了一种本文讨论的上扬方案之外的替代方案: 标准模型 sector 的量子效应可以将真空能足够小的反德西特真空抬升。

Here

此处

$$D_{\alpha} = i\mathcal{G}_m X_{\alpha}^m,\ \mathcal{G} = K + \log|W|^2, \tag{132}$$

and X_{α}^m define the Killing vectors such that under a gauge transformation, the matter fields ψ_m transform as $\delta\psi^m = X_{\alpha}^m \varepsilon^{\alpha}$. For linearly realized gauge symmetry, $X_{\alpha}^m = (-iT_{\alpha})_k^m \psi^k$.

且 X_{α}^m 定义了基林矢量，满足规范变换下物质场 ψ_m 按 $\delta\psi^m = X_{\alpha}^m \varepsilon^{\alpha}$ 变换。对于线性实现的规范对称性，有 $X_{\alpha}^m = (-iT_{\alpha})_k^m \psi^k$ 。

Since the D-term potentials are manifestly positive, unlike F-terms, it is natural to inquire if they can be used for de Sitter uplift [30]. In generic orientifold constructions, D-terms that can potentially lead to de Sitter vacua can be identified. The point is that the O7-plane fixed loci carry D7-brane charge and so require the presence of D7-branes to cancel D7-brane tadpoles. Stacks of D7-branes wrapping a divisor D_a host a corresponding gauge theory, with gauge coupling g_a determined by $\langle \mathrm{Re}\, T_a \rangle = 1/g_a^2$. The D7-brane Chern-Simons coupling

由于 D 项势能显然为正，和 F 项不同，人们自然会探究能否用 D 项实现德西特上扬 [30]。在一般定向模构造中，可以识别出有可能产生德西特真空的 D 项。关键在于，O7 平面的不动轨迹带有 D7 膜荷，因此需要存在 D7 膜来抵消 D7 膜蝌蚪。堆叠的 D7 膜卷积分除子 D_a 后会对应一个规范理论，其规范耦合 g_a 由 $\langle \mathrm{Re}\, T_a \rangle = 1/g_a^2$ 确定。D7 膜陈-西蒙斯耦合

$$\int_{\mathcal{M}_8} C_4 \wedge \mathrm{Tr}\, e^{\frac{iF}{2\pi}} \tag{133}$$

gives rise to a $\hat{B} \wedge F$ term in four dimensions, with $\hat{B}_{\mu\nu} \propto C_{\mu\nu mn}$, after expanding the exponential and turning on magnetic fluxes $\langle F^{mn} \rangle \neq 0, m, n = 4, \ldots, 7$. The $\hat{B} \wedge F$ term, in turn, leads to the cross term of a Stückelberg Lagrangian $\left(A_{\mu} + \partial_{\mu} a\right)^2$, with a the axion component of T_a (dual to the antisymmetric tensor in the sense that $\partial^{\mu} a = \varepsilon^{\mu\upsilon\rho\sigma} \partial_{\upsilon} \hat{B}_{\rho\sigma}$), which implies that the Kähler potential K depends on the combination

$T+\bar{T}+q\widehat{V}$. This Kähler potential gives rise to a (field-dependent) Fayet-Iliopoulos (FI) D-term proportional to $\partial K/\partial\widehat{V}|_{\widehat{V}=0}$. This can be seen as the modulus T_a having a charge q under the corresponding $U(1)$ symmetry.

在四维中产生一个 $\widehat{B}\wedge F$ 项,其中 $\widehat{B}_{\mu\nu} \propto C_{\mu\nu mn}$,这是展开指数并打开磁通量 $\langle F^{mn}\rangle \neq 0, m, n = 4, \dots, 7$ 后得到的。该 $\widehat{B} \wedge F$ 项反过来会导出施蒂克尔贝格拉格朗日量的交叉项 $\left(A_\mu + \partial_\mu a\right)^2$ ，其中 a 是 T_a 的轴子分量 (在 $\partial^\mu a = \varepsilon^{\mu\upsilon\rho\sigma}\partial_\upsilon\widehat{B}_{\rho\sigma}$ 的意义下对偶于反对称张量)，这意味着凯勒势 K 依赖于组合 $T+\bar{T}+q\widehat{V}$ 。该凯勒势产生正比于 $\partial K/\partial\widehat{V}|_{\widehat{V}=0}$ 的 (依赖场的) 费耶特-伊利亚普洛斯 (FI)D 项。这可以理解为模场 T_a 在对应 $U(1)$ 对称性下带电荷 q 。

In general, FI terms ξ_{FI} contribute to a $U(1)$ D-term potential as

一般而言，FI 项 ξ_{FI} 对 $U(1)$ D 项势的贡献为

$$V_D \propto \left(\xi_{FI} - \sum_m q_m K_m \varphi_m\right)^2, \tag{134}$$

where q_m are the charges of matter fields φ_m . Since V_D is positive-definite, the D-term potential (134) may uplift the F-term scalar potential unless the minimum of (134) occurs at $V_D = 0$.

其中 q_m 是物质场 φ_m 的电荷。由于 V_D 是正定的，除非 (134) 的极小值出现在 $V_D = 0$ 处，否则 D 项势 (134) 可以抬升 F 项标量势。

Unfortunately, as can be seen from (132), in supergravity the vanishing of the F-term ($F_m \propto D_m W$) implies the vanishing of the D-term. So the D-term uplift mechanism does not work for KKLT [178,179], since the AdS vacuum is supersymmetric. This mechanism may work in LVS since in that case the AdS vacuum is not supersymmetric, and so the F-term is nonzero. In explicit string realizations, the D7-branes support not just gauge fields but also matter fields [36] , naturally leading to the T-brane mechanism that we discuss next (see also [181, 182] for related approaches).

遗憾的是，从式 (132) 可以看出，在超引力中 F 项 ($F_m \propto D_m W$) 为零意味着 D 项也为零。因此 D 项抬升机制对 KKLT 不起作用 [178,179]，因为 AdS 真空是超对称的。该机制可以在 LVS 中生效，因为在这种情况下 AdS 真空不是超对称的，因此 F 项非零。在具体的弦论实现中，D7 膜不仅承载规范场，也承载物质场 [36] ，自然导出我们接下来讨论的 T 膜机制 (相关研究参见 [181, 182])。

- T-brane uplift. If supersymmetry is broken, the dependence of the scalar potential on the matter fields φ_m will include not only the D-term contribution V_D but also the contribution from soft supersymmetry breaking terms, such as scalar masses:

- T 膜抬升。如果超对称性破缺，标量势对物质场 φ_m 的依赖不仅包含 D 项贡献 V_D ，还包含软超对称破缺项的贡献，例如标量质量:

$$V_{\text{soft}} = \sum_n m_n^2 |\varphi_n|^2. \tag{135}$$

Then minimizing the potential $V_D + V_{\text{soft}}$ with respect to φ_n will lead to a positive contribution to the overall scalar potential, since both terms cannot vanish simultaneously. The resulting positive energy can cause uplifting, with $V_{\rm up} \sim |W_0|^2/\mathcal{V}^{8/3}$ [183]. In this sense the uplifting comes from a combination of D-terms and F-terms.

随后对 φ_n 极小化势 $V_D + V_{\rm soft}$ 会给总标量势带来正贡献，因为两项不可能同时为零。由此产生的正能量可以实现抬升，其中 $V_{\rm up} \sim |W_0|^2/\mathcal{V}^{8/3}$ [183]。在这个意义上，抬升来自 D 项和 F 项的共同作用。

This mechanism has an interesting string theory realization in terms of magnetic and three-form fluxes as well as T-branes [183]. T-branes are D-brane configurations or bound states defined by the fact that the Higgs fields Φ in the adjoint representation of the brane non-abelian gauge group, written in a matrix form, are not diagonalizable. Thus, $\left[\Phi, \Phi^\dagger\right] \neq 0$, and Φ can be written not in a diagonal but in a triangular form, hence the term T-brane [184-186].

该机制在弦论中可以通过磁通量、三形式场以及 T 膜得到有趣的实现 [183]。T 膜是满足如下条件的 D 膜构型或束缚态: 写成矩阵形式后，膜非阿贝尔规范群伴随表示中的希格斯场 Φ 不可对角化。因此，$\left[\Phi, \Phi^\dagger\right] \neq 0$ ，且 Φ 不需要写成对角形式，而可以写成三角形式，“T 膜”因此得名 [184-186]。

The eight-dimensional equations of motion for an orientifolded stack of N D7-branes are

定向投影后 N 张 D7 膜的八维运动方程为

$$J \wedge \mathcal{F} - \left[\Phi, \Phi^\dagger\right] = 0, \tag{136}$$

where J is the pullback of the Kähler form, the two-form flux is $\mathcal{F} = (2\pi\alpha')\ F - \iota^* B$ with $\iota^* B$ the pullback of the NS-NS B-field, and Φ is the scalar field in the adjoint representation of the corresponding $U(N)$ gauge group. Equation (136) corresponds to the four-dimensional D-term equation with Φ the canonically normalized field. If the flux $\mathcal{F}$ is non-primitive, i.e., if $J \wedge \mathcal{F} \neq 0$, then $\left[\Phi, \Phi^\dagger\right] \neq 0$, and hence Φ cannot be a diagonal matrix. On the other hand, a triangular matrix satisfies this condition, thus defining the D-brane configuration as a T-brane.

其中 J 是凯勒形式的拉回，二形式场强为 $\mathcal{F} = (2\pi\alpha')\ F - \iota^* B$ ，$\iota^* B$ 是 NS-NS B 场的拉回，Φ 是对应 $U(N)$ 规范群伴随表示中的标量场。方程 (136) 对应四维 D 项方程，其中 Φ 是正则归一化场。若场强 $\mathcal{F}$ 非本原，即满足 $J \wedge \mathcal{F} \neq 0$ ，则可得 $\left[\Phi, \Phi^\dagger\right] \neq 0$ ，因此 Φ 不可能是对角矩阵。另一方面，三角矩阵满足该条件，因此这类 D 膜构型被定义为 T 膜。

Furthermore the nonvanishing $(0,3)$ fluxes break supersymmetry, and the soft masses $m^2|\Phi|^2$ can be computed explicitly from the D7-brane DBI action, leading to a stringy realization of the T-brane uplift scenario [183].

此外，非零的 $(0,3)$ 场强会破缺超对称，软质量 $m^2|\Phi|^2$ 可从 D7 膜的 DBI 作用量显式计算得到，由此得到弦论中 T 膜上抬场景的实现 [183]。

[36] Chiral matter on D7-branes, for example from a realization of the MSSM, can interfere with moduli stabilization [180].

36 D7 膜上的手征物质 (例如最小超对称标准模型 MSSM 的实现中产生的手征物质) 会对模稳定产生干涉 [180]。

- Complex structure moduli uplift. As we have seen, the complex structure moduli part of the scalar potential V_{cs} is positive semi-definite. It is then naturally minimized at the supersymmetric point $D_{z_i} = 0$ with $V_{\mathrm{cs}} = 0$. However, considering the full scalar potential and knowing that V_{cs} is also a function of the Kähler moduli and that the rest of the scalar potential is not positive, V_{cs} can be considered as an uplift term [187-189].

- 复结构模上抬。如前所述，标量势的复结构模部分 V_{cs} 是半正定的。它自然在超对称点 $D_{z_i} = 0$ (满足 $V_{\mathrm{cs}} = 0$) 处取得极小值。然而，考虑完整标量势且已知 V_{cs} 同时也是凯勒模的函数，且标量势的其余部分非正定，因此 V_{cs} 可被视作上抬项 [187-189]。

This can be seen clearly in LVS, where $V_{\mathrm{cs}} \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$ whereas $V_{\mathrm{K}} \sim \mathcal{O}\left(1/\mathcal{V}^3\right)$. Therefore, instead of concluding that positivity implies $V_{\mathrm{cs}} = 0$, we should say that positivity implies $V_{\mathrm{cs}} \sim \mathcal{O}\left(1/\mathcal{V}^3\right)$, and if V_{cs} is positive, it could play the role of an uplifting term.

这一点在大体积场景 (LVS) 中十分清晰，其中 $V_{\mathrm{cs}} \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$ 而 $V_{\mathrm{K}} \sim \mathcal{O}\left(1/\mathcal{V}^3\right)$ 。因此，我们不应得出正性蕴涵 $V_{\mathrm{cs}} = 0$ 的结论，而是应当说正性蕴涵 $V_{\mathrm{cs}} \sim \mathcal{O}\left(1/\mathcal{V}^3\right)$ ，若 V_{cs} 为正，它就可以起到上抬项的作用。

This proposal requires a large amount of tuning to get the desired size for V_{cs} , but such tuning may be possible given the large number of fluxes. A concrete example using the continuous flux approximation was presented in [188].

该方案需要对 V_{cs} 的期望尺寸进行大量调谐，但考虑到场的数量众多，这种调谐是可能实现的。文献 [188] 给出了使用连续场近似的具体例子。

- Non-perturbative dilaton superpotential uplift. In type IIB string theory, matter fields can be located either on D3-branes or D7-branes, which can host either the Standard Model or a hidden sector. In both KKLT and LVS, we have considered hidden sectors on D7-branes, for which the gauge coupling g_a is determined by the volume Re T_a of the divisor D_a wrapped by the D7- branes, and the non-perturbative superpotential terms W_{np} are proportional to e^{-cT_a} . However, hidden sectors may also be located on D3-branes, in which case the gauge coupling is determined by the axiodilaton τ . The non-perturbative superpotential is then of the form:

- 非微扰 dilaton 超势上抬。在 IIB 型弦论中，物质场可以位于 D3 膜或 D7 膜上，这些膜可以承载标准模型或隐藏域。在 KKLT 场景和 LVS 场景中，我们都讨论了 D7 膜上的隐藏域，其规范耦合 g_a 由 D7 膜缠绕的除子 D_a 的体积 Re T_a 决定，非微扰超势项 W_{np} 正比于 e^{-cT_a} 。然而，隐藏域也可以位于 D3 膜上，这种情况下规范耦合由轴 dilaton τ 决定，此时非微扰超势的形式为:

$$W_{\mathrm{np}} = B(z_i, \tau)\, e^{-b(\tau-\rho)} \tag{137}$$

where ρ is the blowup of the singularity where D3-branes are located. After fixing ρ at the singularity $\langle\rho\rangle = 0$ from its D-term field equations and also using $\langle\tau\rangle$ from the flux superpotential [37] , the term (137) makes a contribution to the total F-term scalar potential of the form [183]:

其中 ρ 是 D3 膜所在奇点的爆胀。将奇点 $\langle\rho\rangle = 0$ 处的 ρ 通过其 D 项场方程固定后，再利用来自通量超势 [37] 的 $\langle\tau\rangle$，式 (137) 对如下形式的总 F 项标量势产生贡献 [183]:

$$V_{\mathrm{up}} = C\frac{e^{-2b\langle\tau\rangle}}{\mathcal{V}} \geq 0 \tag{138}$$

with C a positive constant. Adding the contribution of these superpotentials to the LVS potential provides a potential uplift term [183]. This proposal is model-dependent and still lacks an explicit string example. It would be interesting to search for such a model since it works in a way very similar to the anti-D3- brane uplift, in the sense that the uplifting term can be easily tuned to match the $\mathcal{O}\left(\mathcal{V}^{-3}\right)$ of the LVS potential, with the non-perturbative term $e^{-2b\langle\tau\rangle}$ playing the role of the warp factor.

其中 C 为正常数。将这些超势的贡献加入 LVS 势后可得到一个抬高势项 [183]。该方案依赖具体模型，目前仍缺少明确的弦论实例。由于其作用方式与反 D3 膜抬升非常相似，寻找这类模型很有意义: 抬升项可以轻松调节以匹配 LVS 势的 $\mathcal{O}\left(\mathcal{V}^{-3}\right)$，而非微扰项 $e^{-2b\langle\tau\rangle}$ 发挥翘曲因子的作用。

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[37] Recall that the dilaton is fixed at leading order by the fluxes, as it appears in the tree-level flux superpotential. Thus, we can replace τ by its vev at this order.

[37] 我们知道，dilaton 在领头阶已被通量固定，因为它出现在树级通量超势中。因此我们可以在该阶将 τ 替换为其真空期望值。

- Flux superpotential uplift. Finally, structure in the flux superpotential itself can lead to spontaneous breaking of supersymmetry in the complex structure moduli and axiodilaton sector. Paired with a non-perturbative superpotential depending on - and serving to stabilize - the Kähler moduli, as in the KKLT scenario, the result can be a metastable de Sitter minimum.

- 通量超势抬升。最后，通量超势本身的结构就可以导致复结构模和轴子 dilaton 部分的超对称自发破缺。如同 KKLT 方案中，配合依赖于凯勒模并用来稳定凯勒模的非微扰超势，最终可以得到一个亚稳态德西特极小值。

Specifically, suppose that in compactification of type IIB string theory on an O3/O7 orientifold of a Calabi-Yau threefold X_6 , three-form fluxes are chosen to produce a PFV, as in (77), and are particularly chosen so that the effective superpotential (83) for the axiodilaton coordinate along the PFV is a sum of N dilogarithms:

具体来说，假设 IIB 型弦论在卡拉比-丘三维流形 X_6 的 O3/O7 定向形上紧致化，如式 (77) 那样选取三形式通量来产生 PFV，且特别选取使得 PFV 方向上轴子 dilaton 坐标的有效超势 (83) 为多个 N 二重对数之和:

$$W_{\text{eff}}(\tau) = -\frac{1}{2^{3/2}\pi^{5/2}}\left(\sum_{I=1}^{N}\mathbf{M}\cdot\beta_I\,\mathrm{GV}(\beta_I)\,\mathrm{Li}_2\left(e^{2\pi i\tau\mathbf{p}\cdot\beta_I}\right)\right)+\ldots, \tag{139}$$

and the omitted terms are subleading. Here $\mathbf{M}$ is a fixed set of fluxes, β_I are effective curve classes of the mirror threefold $\widetilde{X}_6$, and $\mathbf{p}$ is a vector in the Kähler cone of $\widetilde{X}_6$. For most choices of $X_6, \mathbf{M}$, and $\mathbf{p}$, the N terms in (139) will not effectively compete to produce a minimum at finite τ . However, for $N > 3$ the structure in (139) can be sufficient to produce a metastable de Sitter minimum (Demirtas et al., Unpublished). This mechanism has not yet been realized in an explicit compactification.

省略的项都是次领头项。这里 $\mathbf{M}$ 是一组固定的通量，β_I 是镜像三维流形 $\widetilde{X}_6$ 的有效曲线类，$\mathbf{p}$ 是 $\widetilde{X}_6$ 凯勒锥中的一个向量。对于大多数 $X_6, \mathbf{M}$ 和 $\mathbf{p}$ 的选取，式 (139) 中的 N 项无法有效竞争，在有限 τ 处产生极小值。但对于 $N > 3$ ，式 (139) 的结构足以产生亚稳态德西特极小值 (Demirtas 等人，未出版)。该机制目前尚未在显式紧致化中实现。

The preceding sections illustrate some of the significant efforts that have been dedicated to achieving moduli stabilization in anti-de Sitter and de Sitter vacua of type IIB string theory (other string theories will be discussed in section ”Beyond IIB”). All of the components of the KKLT and LVS scenarios have undergone continuing testing in extremely detailed realizations. Critical scrutiny has been directed at anti-D3-brane supersymmetry breaking (section ”Anti-D3-Branes”), the ten-dimensional description of non-perturbative effects (section ”Ten-Dimensional Description”), and corrections that may ruin the validity of various approximations, among many other issues. There have even been speculations that string theory may have no de Sitter solutions [190, 191]. For a sample of recent work emphasizing and addressing these challenges, see [105, 106, 117, 118, 120, 122-125, 144, 148, 150, 151, 155, 163, 192-212]. The intensity and focus of the recent literature illustrates that the field is very much alive and active.

前面几节说明了，为了在 IIB 型弦论的反德西特和德西特真空中实现模稳定，学界已经付出了诸多重要努力 (其他弦论将在“IIB 之外”一节讨论)。KKLT 和 LVS 方案的所有组成部分都在极为细致的构造中持续接受检验。对反 D3 膜超对称破缺 (“反 D3 膜”一节)、非微扰效应的十维描述 (“十维描述”一节)，以及可能破坏各类近似有效性的修正等诸多问题，学界都进行了严格审查。甚至有猜想认为弦论可能不存在德西特解 [190, 191]。关于近期强调并解决这些挑战的研究实例，参见 [105, 106, 117, 118, 120, 122-125, 144, 148, 150, 151, 155, 163, 192-212]。近期文献的研究热度与聚焦方向表明，该领域目前依然充满活力，十分活跃。

Moduli Stabilization and the Standard Model

模稳定与标准模型

As the examples above illustrate, there are many approaches to the problems of stabilizing moduli and obtaining metastable de Sitter vacua in type IIB string theory. However, finding a de Sitter vacuum is not the final goal: to describe quantum gravity in our Universe, we need a de Sitter solution that also incorporates the Standard Model.

正如上述例子所示，在 IIB 型弦论中，解决模稳定和获得亚稳德西特真空这两个问题存在多种方法。但找到德西特真空并非最终目标: 要描述我们宇宙中的量子引力，我们需要一个同时包含标准模型的德西特解。

The tasks of moduli stabilization and of embedding realistic particle physics in string theory are intertwined. The gauge and Yukawa couplings of the Standard Model particles, and the masses of their superparters, are free parameters in the EFT, and it is the dynamics of moduli stabilization that fixes their values. Thus, moduli stabilization is a prerequisite for extracting meaningful four-dimensional particle physics from string compactifications. At the same time, the presence of chiral matter can impact moduli stabilization [180].

模稳定与在弦论中嵌入现实粒子物理这两项任务是相互交织的。标准模型粒子的规范耦合、汤川耦合，以及它们超对称伙伴的质量，在有效场论中都是自由参数，正是模稳定的动力学确定了这些参数的取值。因此，模稳定是从弦紧致化中提取有意义的四维粒子物理的前提。与此同时，手征物质的存在也会影响模稳定 [180]。

A full treatment of particle physics in string theory is beyond the scope of this work (see for instance [213] for a recent review, and [214] for lectures on possible signatures). However, we can mention several concrete examples of quasi-realistic Calabi-Yau orientifold models with stabilized moduli [215-219]. These models are examples of the modular or bottom-up approach to string model building [220-223], which separates local questions, such as the realization of the Standard Model on a set of D-branes, from global issues such as moduli stabilization and supersymmetry breaking. Each question can be addressed separately, and at the end the two can be put together into consistent Calabi-Yau compactifications.

对弦论中的粒子物理做完整讨论超出了本文的范围 (近期综述可参见例如 [213]，相关唯象信号介绍可参见 [214])。不过，我们可以举出数个已完成模稳定的准现实卡拉比-丘定向折叠模型的具体例子 [215-219]。这些模型是弦模型构建中模块化方法即自下而上方法的实例 [220-223]，该方法将局域问题 (例如在一组 D 膜上实现标准模型) 与全局问题 (例如模稳定和超对称破缺) 分离开来: 两类问题可以分别处理，最终再将二者结合为自洽的卡拉比-丘紧致化。

Even though considerable success has been achieved in this direction, it is fair to say that much more work will be required to establish well-controlled de Sitter vacua in string theory or F-theory models with realistic particle physics.

尽管我们已经在这一方向取得了可观的进展，但客观地说，要在含现实粒子物理的弦论或 F-理论模型中得到控制良好的德西特真空，仍需要大量更多的研究工作。

Soft Supersymmetry-Breaking Terms

软超对称破缺项

Supersymmetry breaking in the moduli sector naturally connects to the physics of the Standard Model. In type IIB string theory, the Standard Model may arise on D3-branes at a singularity, or on D7-branes wrapping four cycles (which can also be generalized to F-theory constructions). Mediation of supersymmetry breaking to the Standard Model sector is model-dependent. However, because the moduli fields originate from the gravitational sector and so have gravitational-strength couplings, moduli-mediated supersymmetry breaking is a variant of gravity-mediated super-symmetry breaking.

模空间的超对称破缺自然与标准模型物理相关联。在 IIB 型弦论中，标准模型可以起源于奇点处的 D3 膜，也可以起源于缠绕四维周期的 D7 膜 (该构造也可推广到 F 理论)。超对称破缺传递到标准模型 Sector 的方式依赖于具体模型。但由于模场源自引力 Sector，因此具有引力强度耦合，模介导的超对称破缺是引力介导超对称破缺的一个分支。

The general structure of soft terms from moduli stabilization has been known for many years (see for instance [224,225]). Writing the gaugino masses as $M_{1/2}$, the scalar masses as m_0 , and the trilinear terms as $A_{\alpha\beta\gamma}$ with indices α, β, γ running over the Standard Model matter fields, one finds:

模稳定产生软项的一般结构已经被研究多年 (例如参见 [224,225])。将 gaugino 质量写为 $M_{1/2}$ ，标量质量写为 m_0 ，三线性项写为 $A_{\alpha\beta\gamma}$ ，下标 α, β, γ 遍历标准模型物质场，可得:

$$M_{1/2} = \frac{1}{f+\bar{f}} F^I \partial_I f$$

$$m_\alpha^2 = V_0 + m_{3/2}^2 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \log \hat{K}_\alpha, \tag{140}$$

$$A_{\alpha\beta\gamma} = F^I K_I + F^I \partial_I \log Y_{\alpha\beta\gamma} - F^I \partial_I \left(\hat{K}_\alpha \hat{K}_\beta \hat{K}_\gamma \right).$$

Here $Y_{\alpha\beta\gamma}$ are the Yukawa couplings of the matter fields; f is the moduli-dependent gauge kinetic function; F^I are the moduli F-terms, with the indices I, J running over the moduli; and $\hat{K}_i$ is the moduli-dependent Kähler potential for the matter fields φ_α . The full Kähler potential $\mathcal{K}$ is the sum of the moduli Kähler potential K and the leading-order matter Kähler potential $\hat{K}_\alpha |\varphi_\alpha|^2$:

此处 $Y_{\alpha\beta\gamma}$ 是物质场的汤川耦合；f 是依赖于模的规范动力学函数；F^I 是模 F 项，下标 I, J 遍历所有模；$\hat{K}_i$ 是物质场 φ_α 依赖于模的凯勒势。完整的凯勒势 $\mathcal{K}$ 是模凯勒势 K 与领头阶物质凯勒势 $\hat{K}_\alpha |\varphi_\alpha|^2$ 之和:

$$\mathcal{K} = K(\tau, T_a, z_i) + \hat{K}_\alpha |\varphi_\alpha|^2 + \ldots \tag{141}$$

The above quantities have been computed in various D-brane configurations representing the Standard Model [170, 178, 226-229]. In the table below, we summarize the order of magnitude of the soft terms for the KKLT and LVS models, taking the Standard Model on D3-branes or D7-branes, assuming anti-D3-brane uplift, and using the nilpotent superfield formalism to capture the breaking of supersymmetry [170] (for a recent discussion using dilaton superpotential uplifting, see [230]). Notice that the generic expectation that soft terms should be of order $m_{3/2}$ does not hold for string models in which there are suppressions either by $1/\log\left(|W_0|^{-1}\right)$ or by powers of $1/\mathcal{V}$.

上述物理量已经在多种代表标准模型的 D 膜构型中完成计算 [170, 178, 226-229]。下表我们总结了 KKLT 模型与 LVS 模型中软项的数量级，取标准模型位于 D3 膜或 D7 膜上，假设采用反 D3 膜抬升，并利用幂零超场形式论描述超对称破缺 [170](关于近期利用 dilation 超势抬升的讨论，参见 [230])。注意，软项量级应为 $m_{3/2}$ 的普遍预期在弦模型中并不成立，这类模型中软项会被 $1/\log\left(|W_0|^{-1}\right)$ 或 $1/\mathcal{V}$ 的幂次压低。

In particular, scenarios such as split supersymmetry, in which the fermionic superpartners (gauginos) are hierarchically lighter than the scalar superpartners (squarks and sleptons), can be obtained from string models. The Standard Model on D3-branes manifests a variant of sequestered supersymmetry breaking, in the sense that the source of supersymmetry breaking couples very weakly to the Standard Model fields and there are usually first-order cancellations for the leading contributions to the soft terms. This is similar to the no-scale structure of the moduli potential in that the nontrivial structure comes from higher-order corrections. However, the corrections to the matter Kähler potential are poorly understood.

特别地，诸如分裂超对称这类场景——费米超对称伙伴 (gaugino) 层级地轻于标量超对称伙伴 (squark 和 slepton)——可以从弦模型中得到。D3 膜上的标准模型呈现出隐匿超对称破缺的一种变体: 超对称破缺的源与标准模型场的耦合极弱，软项领头阶贡献通常存在一阶抵消。这和模势的无标度结构类似，非平凡结构来自高阶修正。但物质凯勒势的修正目前还研究得很不充分。

In general the soft terms depend on the location of the Standard Model (D3- brane or D7-branes), on the de Sitter uplifting mechanism, and on corrections to the matter Kähler potential. For instance, in LVS with the Standard Model on D3- branes at singularities, the masses in Planck units are as follows: the gravitino mass is $m_{3/2} \sim \mathcal{O}(1/\mathcal{V})$; the gaugino masses are further suppressed, $M_{1/2} \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$; and scalar masses can be $m_0 \sim \mathcal{O}\left(1/\mathcal{V}^{3/2}\right)$ or $m_0 \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$. In the first case, it is clear that $m_{3/2} \gg m_0 \gg M_{1/2}$, which is a typical case of split supersymmetry. The second case would then correspond to high-scale supersymmetry breaking.

一般来说，软破缺项取决于标准模型的位置 (D3 膜或 D7 膜)、德西特 uplift 机制，以及对物质凯勒势的修正。例如，在奇点处 D3 膜上承载标准模型的大体积紧致化 (LVS) 中，普朗克单位下的质量如下: 引力微子质量为 $m_{3/2} \sim \mathcal{O}(1/\mathcal{V})$；戈迪诺质量进一步压低，为 $M_{1/2} \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$；标量质量可以是 $m_0 \sim \mathcal{O}\left(1/\mathcal{V}^{3/2}\right)$ 或 $m_0 \sim \mathcal{O}\left(1/\mathcal{V}^2\right)$。第一种情况中，显然满足 $m_{3/2} \gg m_0 \gg M_{1/2}$ ，这是分裂超对称的典型情形。第二种情况则对应高标度超对称破缺。

The hierarchies can be reduced by several sources of de-sequestering [231-233]. De-sequestering can occur if flavor D7-branes intersect the D3-brane singularity, or if the singularity is orientifolded (and not just orbifolded) [38] . Furthermore, superpotential de-sequestering can be caused by couplings of the form [231]:

层级可以通过多种去隔离源降低 [231-233]。当味 D7 膜与 D3 膜奇点相交,或者奇点经过了 orientifold 投影 (不只是轨道投影) 时，就会发生去隔离 [38] 。此外，超势去隔离可由如下形式的耦合引发 [231]:

$$\Delta W = \mathcal{O}_{\text{vis}}\, e^{-2\pi T}, \tag{142}$$

where $\mathcal{O}_{\text{vis}}$ is a gauge-invariant chiral operator constructed from visible sector chiral superfields and T is a Kähler modulus. The coupling (142) can be viewed as a result of closed string exchange: thus, de-sequestering can occur even if the visible sector D-branes and the four-cycle supporting Euclidean D3-branes do not intersect. See [230] for an up-to-date discussion of sequestering in LVS.

其中 $\mathcal{O}_{\text{vis}}$ 是由可见区手征超场构造的规范不变手征算符，T 是一个凯勒模。耦合式 (142) 可以被视为闭弦交换的结果: 因此，即使可见区 D 膜与承载欧几里得 D3 膜的四周期不相交，也会发生去隔离。关于 LVS 中隔离问题的最新讨论参见 [230]。

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[38] In this case it has been argued that moduli redefinitions are needed at one loop that would cause de-sequestering of soft terms [73].

[38] 已有研究指出，这种情况下单圈阶需要模重定义，这会引发软项去隔离 [73]。

Beyond IIB

非 IIB 型

There is no fundamental reason to restrict to compactifications of type IIB string theory as an arena for studying moduli stabilization, and as we will soon see, there are very interesting results in other string theories and in M-theory. However, some practical and technical considerations are responsible for the fact that type IIB string is overrepresented in the last 20 years of work on moduli stabilization. Let us briefly note these causes.

并没有根本性理由要求我们只能将 IIB 型弦论紧化作为研究模稳定的研究场景，我们很快就会看到，在其他弦论和 M 理论中也存在非常有意思的结论。然而，一些实操层面和技术层面的考量导致过去 20 年模稳定研究中 IIB 型弦论占比过高。我们来简要列出这些原因。

The first reason is the fact that the leading backreaction of the fluxes on the metric is only a scaling of the original Calabi-Yau metric: one can find conformally Calabi-Yau solutions (22) in type IIB flux compactifications [17]. This allows one to extend the use of Calabi-Yau geometry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ solutions.

第一个原因是，通量对度规的主要反作用仅会缩放原卡拉比-丘度规: 在 IIB 型通量紧化中可以找到共形卡拉比-丘解 [17](22)，这使得我们可以将卡拉比-丘几何的应用从 $\mathcal{N} = 2$ 解拓展到 $\mathcal{N} = 1$ 解。

A second reason is that arguably the most robust mechanism for breaking supersymmetry at a parametrically low scale [97] involves an exponentially warped throat region that caps off smoothly in the infrared: this is in contrast to solutions with singularities, in which the supersymmetry breaking is incalculable. At the time of writing, the Klebanov-Strassler solution [96] of type IIB supergravity (and close relatives involving orbifolding or deforming it) is the only known solution with these properties. Changing this state of affairs would be an important advance.

第二个原因是，目前公认在参数低标度下破缺超对称最可靠的机制 [97] 依赖一个在红外端平滑封顶的指数翘化喉区: 这一点和带奇点的解不同，后者的超对称破缺无法计算。截至目前，IIB 型超引力的克莱班诺夫-斯特拉斯勒解 [96](以及它经过轨道形变换或形变得到的近亲解) 是唯一已知满足上述性质的解。改变这一现状会是一项重要进展。

A third reason is that in some of the other corners of the duality web, constructing solutions with $\mathcal{N} = 1$ supersymmetry involves significantly greater mathematical challenges. For example, M-theory compactifications on G_2 manifolds require working in real differential geometry, while for F-theory compactifications on Calabi-Yau fourfolds, the Kähler potential is poorly understood.

第三个原因是，在对偶网的其他部分中，构造带有 $\mathcal{N} = 1$ 超对称的解会带来大得多的数学挑战: 例如，M 理论在 G_2 流形上紧化需要研究实微分几何，而 F 理论在卡拉比-丘四元流形上紧化时，人们对其凯勒势的理解还十分有限。

These observations should be understood as explanations for the state of the literature, or indeed as calls to arms, not as no-go results. With this understanding, let us proceed to briefly survey developments in other string theories.

这些观察只是用来解释现有研究文献的现状，甚至算是呼吁大家开展研究，而绝非否定性结论。明确这一点后，我们来简要综述其他弦论中的相关进展。

Type IIA Flux Compactifications

IIA 型通量紧化

In type IIB string theory, non-perturbative effects have proved necessary to stabilize all the moduli [39], but in type IIA string theory, all moduli can be stabilized with fluxes alone. The best-understood construction is due to DeWolfe, Giryavets, Kachru, and Taylor (DGKT) [234]. Let us briefly review the main points of this scenario.

在 IIB 型弦论中，已经证明要稳定所有模必须引入非微扰效应 [39]，但在 IIA 型弦论中，仅靠通量就可以稳定所有模。研究最透彻的构造由 DeWolfe、Giryavets、Kachru 和 Taylor(DGKT) 完成 [234]。下面我们简要概述这一方案的核心内容。

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[39] As explained in section ”LVS”, cf. (97), purely perturbative stabilization is a logical possibility in type IIB flux compactifications, but has not yet been achieved in a controlled example.

[39] 如“大体积情景 (LVS)”一节中 (97) 式所述，纯微扰稳定在 IIB 型通量紧化中是逻辑上可行的，但目前尚未得到可控的例子。

The starting point is the bosonic ten-dimensional action for massive type IIA supergravity with massless Neveu-Schwarz-Neveu-Schwarz bosonic fields including the metric g_{MN}, the dilaton ϕ and the antisymmetric tensor B_{MN}, with field strength $H_3 = \mathrm{d}B_2$, and Ramond-Ramond fields C_1, C_3 with field strengths F_2, F_4, as well as the Romans mass term F_0.

我们的出发点是质量型 IIA 超引力的玻色十维作用量，其中包含无质量的 NS-NS 玻色场: 度规 g_{MN}
、dilaton ϕ 和场强为 $H_3 = \mathrm{d}B_2$ 的反对称张量 B_{MN} ，还有 Ramond-Ramond 场 C_1, C_3 及其场强 F_2, F_4
，以及 Romans 质量项 F_0 。

Compactifying massive type IIA supergravity on the orientifold $T^6/\mathbb{Z}_3^2$ with ori-entifold action $\mathcal{O} = \Omega_{\mathrm{ws}}(-1)^{F_L}\sigma$, with Ω_{ws} worldsheet parity, $(-1)^{F_L}$ left-moving fermion number, and σ an involution acting on the three complex coordinates x_i as $\sigma : x_i \to -\bar{x}_i$, one finds a four-dimensional model with $\mathcal{N} = 1$ supersymmetry and an O6-plane filling the four-dimensional spacetime and wrapping a three-cycle of T^6 . The geometric moduli of this toroidal orientifold are the three untwisted moduli and nine blow-up modes.

将质量型 IIA 超引力在定向轨形 $T^6/\mathbb{Z}_3^2$ 上紧化，定向模作用为 $\mathcal{O} = \Omega_{\mathrm{ws}}(-1)^{F_L}\sigma$ ，其中 Ω_{ws} 是世界面宇称，$(-1)^{F_L}$ 是左行费米子数，σ 是作用在三个复坐标 x_i 上的对合，作用方式为 $\sigma : x_i \to -\bar{x}_i$
，最终得到的四维模型具有 $\mathcal{N} = 1$ 超对称，一个 O6 膜充满四维时空，并且包裹 T^6 的一个三周期。这种环面定向轨形的几何模包括三个非扭模和九个爆破模。

One can include fluxes F_0, F_2, F_4 satisfying the quantization condition:

我们可以引入满足量子化条件的通量 F_0, F_2, F_4 :

$$\int F_q = (2\pi)^{q-1}\alpha'^{(q-1)/2} f_q,\ f_q \in \mathbb{Z}\ q = 0, 2, 4, \tag{143}$$

with a similar expression for H_3 in terms of integers h_3 . To simplify matters we set $f_2 = 0$, since f_2 is not essential for moduli stabilization, and we define the parameters:

H_3 可用整数 h_3 写出类似表达式。为简化讨论，我们设 $f_2 = 0$ ，因为 f_2 对模稳定不是必要的，然后我们定义参数:

$$m_0 = \frac{f_0}{2\pi\sqrt{2\alpha'}},\ p = (2\pi)^2\alpha' h_3,\ e^i = \frac{\kappa^{1/3}}{\sqrt{2}}\left(2\pi\sqrt{\alpha'}\right)^3 f_4^i, \tag{144}$$

with the index $i = 1, 2, 3$ labeling the different four cycles of T^6 . Here κ stands for the normalization of the triple intersection number $\kappa = \int \omega_1 \wedge \omega_2 \wedge \omega_3$, with $\omega_1, \omega_2, \omega_3$ a basis of the T^6 two forms.

其中指标 $i = 1, 2, 3$ 标记 T^6 的不同四周期。这里 κ 代表三重相交数 $\kappa = \int \omega_1 \wedge \omega_2 \wedge \omega_3$ 的归一化，$\omega_1, \omega_2, \omega_3$ 是 T^6 二形式的一组基。

The presence of the O6-plane gives rise to a tadpole that can be cancelled by H_3 and F_0 fluxes satisfying

O6 膜的存在会产生蝌蚪，可由满足下式的 H_3 和 F_0 通量抵消

$$m_0 p = -2\pi\sqrt{2\alpha'}, \tag{145}$$

with no conditions on the F_4 fluxes f_4^i .

且对 F_4 通量 f_4^i 没有约束。

The four-dimensional fields include the dilaton ϕ and the untwisted moduli T_i , which we write as

四维场包含 dilaton ϕ 和非扭模 T_i ，我们将其写为

$$T_i = b_i + it_i, \ \tau = \xi - ie^{-D}, \tag{146}$$

with the volume and four-dimensional dilaton given by

体积和四维 dilaton 由下式给出

$$\mathcal{V} = \kappa_{ijk} t_i t_j t_k =: \kappa t_1 t_2 t_3, \ e^D = \frac{e^\phi}{\sqrt{\mathcal{V}}}, \tag{147}$$

and with axionic fields $C_3 = \sqrt{2}\xi \operatorname{Re}\Omega$ and $B_2 = b_i \omega_i$.

以及轴子场 $C_3 = \sqrt{2}\xi \operatorname{Re}\Omega$ 和 $B_2 = b_i \omega_i$ 。

Incorporating the fluxes in the ten-dimensional action, one finds a nontrivial scalar potential. The axions are minimized at

将通量纳入十维作用量后，我们得到一个非平凡的标量势。轴子在以下位置取得极小值

$$b_i = 0, \ \xi = e_0/p, \ \text{ with } e_0 = \int F_6. \tag{148}$$

The scalar potential for the dilaton and the untwisted moduli is then

dilaton 和非扭模的标量势随后可写为

$$V = \frac{p^2 e^{2\phi}}{4\mathcal{V}^2} + \frac{1}{2}\left(\sum_{i=1}^{3} e_i^2 t_i^2\right)\frac{e^{4\phi}}{2\mathcal{V}} + \frac{m_0^2 e^{4\phi}}{2\mathcal{V}} - \sqrt{2}\,|m_0 p|\,\frac{e^{3\phi}}{\mathcal{V}^{3/2}}. \tag{149}$$

The four terms come from $|H_3|^2, |F_4|^2, |F_0|^2$, and the orientifold, respectively. The potential is simple enough to allow an analytic solution for the minimum:

这四项分别来自 $|H_3|^2, |F_4|^2, |F_0|^2$ 和定向轨形。该势足够简单，可以解析求解极小值:

$$t_i = \frac{1}{|e_i|}\sqrt{\frac{5}{3}\left|\frac{e_1 e_2 e_3}{\kappa m_0}\right|}, \ e^D = |p|\sqrt{\frac{27}{160}\left|\frac{\kappa m_0}{e_1 e_2 e_3}\right|}. \tag{150}$$

The minimum can also be found using the $\mathcal{N} = 1$ effective field theory, with the Kähler potential:

也可以利用 $\mathcal{N} = 1$ 有效场论找到该极小值，对应的 Kähler 势为:

$$K = -\log \mathcal{V} - 4\log\left(i\left(\tau - \overline{\tau}\right)\right). \tag{151}$$

The flux superpotential is

流量超势为

$$W = -p\tau + e_0 + \int J_c \wedge F_4 - \frac{1}{2}\int J_c \wedge J_c \wedge F_2 - \frac{F_0}{6}\int J_c \wedge J_c \wedge J_c,$$

$$= -p\tau + e_0 + e_i T^i + \frac{1}{2}\kappa_{ijk} m^i T^j T^k - \frac{m_0}{6}\kappa_{ijk} T^i T^j T^k, \tag{152}$$

where J_c is the complexified Kähler form $J_c = B_2 + iJ = \sum_{i=1}^{h^{1,1}} T_i \omega_i$. The parameters m^i are proportional to the F_2 fluxes, which we have set to zero. The blow-up modes can be incorporated by writing the volume as

其中 J_c 是复化 Kähler 形式 $J_c = B_2 + iJ = \sum_{i=1}^{h^{1,1}} T_i \omega_i$ 。参数 m^i 与 F_2 流量成正比，我们已将其置零。胀模可通过将体积写为如下形式纳入考量

$$\mathcal{V} = \kappa t_1 t_2 t_3 + \beta \sum_{A=4}^{12} t_A^3. \tag{153}$$

Using (151) and (152) in (21), one recovers (150). The minimum for the blow-up modes can be computed giving

将 (151) 和 (152) 代入 (21)，即可得到 (150)。计算可得胀模的极小值为

$$t_A = -\sqrt{\frac{-10 f_A}{3\beta m_0}} \tag{154}$$

where f_A is the F_4 flux on the corresponding blown-up four-cycle. Provided that $f_A \ll e_i$, the blow-up modes are small enough to justify the toroidal orientifold approximation, in the sense that expansion around the singularity is justified, with the blow-up modes hierarchically smaller than the string scale. The negative signs of the t_A solutions are determined by the Kähler cone conditions. Comparing (153) and (114), we see that the geometry has a Swiss cheese form.

其中 f_A 是对应胀大四维闭链上的 F_4 流量。只要满足 $f_A \ll e_i$ ，胀模就足够小，足以证明环面定向形近似的合理性——也就是说，在奇点附近展开是合理的，胀模按层级小于弦标度。t_A 解的负号由 Kähler 锥条件确定。对比 (153) 和 (114)，可发现该几何具有瑞士奶酪结构。

Let us summarize key features of the DGKT model:

我们来总结 DGKT 模型的核心特征:

- The value of the potential at the minimum is negative:

- 势能在极小值处的取值为负:

$$V = -2\sqrt{\left|\frac{m_0 e_1 e_2 e_3}{15\kappa}\right|} e^{4D} \tag{155}$$

so the solution is an AdS vacuum.

因此该解是 AdS 真空。

- Contrary to the IIB case, the fluxes e_i are not bounded by tadpole conditions, and so the number of solutions is infinite.

- 与 IIB 情形不同，流量 e_i 不受 tadpole 条件约束，因此解的数量是无穷多的。

- The solutions can be supersymmetric or non-supersymmetric [234,235].

- 解可以是超对称的，也可以是非超对称的 [234,235]。

- Scaling the fluxes $e_i \to \lambda e_i$ yields $\mathcal{V} \to \lambda^{3/2}\mathcal{V}, e^{\phi} \to \lambda^{-3/4}e^{\phi}$ and $V \to \lambda^{-9/2}V$, which shows that there is a hierarchy of scales: the larger the fluxes, the larger the volume and the smaller the string coupling.

- 对流量 $e_i \to \lambda e_i$ 标度可得 $\mathcal{V} \to \lambda^{3/2}\mathcal{V}, e^{\phi} \to \lambda^{-3/4}e^{\phi}$ 和 $V \to \lambda^{-9/2}V$ ，这说明存在标度层级: 流量越大，体积越大，弦耦合越小。

In the DGKT solution, the effects of the orientifold planes are accounted for in the smeared approximation. The corresponding localized solutions were studied in [236, 237], to first order in an expansion around the large flux limit.

在 DGKT 解中，定向面的效应在涂抹近似中得到处理。对应局域解已在文献 [236, 237] 中于大流量极限附近的一阶展开下得到研究。

The potential CFT dual of the DGKT model has an interesting property: the operators dual to some of the stabilized moduli have integer conformal dimensions [238-241] (see also the recent extension [242], as well as [243] for similar considerations in the type IIB mirror).

DGKT 模型的对偶势共形场论有一个有趣的性质: 部分已稳定模对应的对偶算符具有整数共形维度 [238-241](另参见近期的推广工作 [242]，以及关于 IIB 镜像类似讨论的 [243])。

Moving beyond the AdS_4 vacua of DGKT, a proposal for classical de Sitter vacua of massive [40] type IIA supergravity, in compactifications on a circle times a negatively curved space, was presented in [245] (see also [246]). These configurations contain localized and backreacted O8-planes, and corrections from string theory are large near the singular sources [41] .

超出 DGKT 的 AdS_4 真空之外，文献 [245] 提出了大质量 [40] IIA 超引力经典德西特真空的一个方案，该方案在圆乘负曲率空间的紧化下成立 (也可参见 [246])。这些构型包含局域化且存在反作用的 O8 平面，弦论修正靠近奇异源 [41] 处很大。

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[40] Massless type IIA supergravity has proved to be a comparatively difficult setting for constructing stabilized vacua. However, see [244], which presented scale-separated vacua of massless type IIA, with controllably small backreaction of the orientifold planes.

[40] 无质量 IIA 超引力一直是构造稳定真空相对困难的框架。不过文献 [244] 给出了无质量 IIA 的标度分离真空，该情形下定向面的反作用可控且很小。

[41] It was argued in [247] that the solutions of [245] are incompatible with the integrated supergravity equations of motion.

[41] 文献 [247] 指出，[245] 的解与积分后的超引力运动方程不相容。

Heterotic Strings

杂弦

The first attempts at moduli stabilization were within the context of the heterotic string. Starting with the $E_8 \times E_8$ string with the Standard Model group inside one of the E_8 factors, the second E_8 provides a hidden sector that could lead to a gaugino condensate superpotential. In the heterotic string, the tree-level gauge kinetic function is $f = S$, where S is the heterotic dilaton field, related to the string coupling by $g_s^{-2} = \langle \mathrm{Re}\, S \rangle$. If the condensing gauge group in the hidden sector is $G \subseteq E_8$, the gaugino condensate superpotential is

模稳定的最早尝试是在杂弦的框架下开展的。从 $E_8 \times E_8$ 弦开始，标准模型群位于其中一个 E_8 因子内部，第二个 E_8 可提供一个隐象限，能够产生戈希诺凝聚超势。在杂弦中，树级规范动力学函数是 $f = S$ ，其中 S 是杂弦 dilation 场，通过 $g_s^{-2} = \langle \mathrm{Re}\, S \rangle$ 与弦耦合关联。若隐象限中的凝聚规范群为 $G \subseteq E_8$ ，则戈希诺凝聚超势为

$$W_{\lambda\lambda} = \mathcal{A} e^{-\frac{8\pi^2}{c(G)} S}. \tag{156}$$

In the presence of H_3 flux, one then has [248, 249] :

存在 H_3 通量时，可得 [248, 249] :

$$W = \int H \wedge \Omega + \mathcal{A} e^{-\frac{8\pi^2}{c(G)} S}. \tag{157}$$

This can be combined with the standard Kähler potential [42] for S and the overall Kähler modulus field T :

可将其与针对 S 和整体凯勒模场 T 的标准凯勒势 [42] 结合:

$$K = -3 \log\left(T + \bar{T}\right) - \log\left(S + \bar{S}\right), \tag{158}$$

to determine the scalar potential for S and T . However, quantization of H_3 fluxes appears to only allow a minimum at strong coupling, beyond the domain of validity of the EFT [249]. Furthermore, the modulus T remains unfixed.

从而确定 S 和 T 的标量势。然而，H_3 通量的量子化似乎仅允许强耦合下的极小值，该区域超出了有效场论 (EFT) 的适用范围 [249]。此外，模 T 仍未被固定。

Three proposals were made to address these problems. First, assuming that the hidden gauge group is a product of at least two gauge groups that allow gaugino condensation, the superpotential for S could be of the racetrack form [251-253]:

学界提出了三个方案来解决这些问题。第一，假设隐规范群是至少两个允许戈希诺凝聚的规范群的乘积，则 S 的超势可以是跑道形式 [251-253]:

$$W(S) = \sum_i \mathcal{A}_i e^{-c_i S} \tag{159}$$

where the exponentials compete with each other to give rise to a weak coupling minimum with $\langle S \rangle \gg 1$
.

其中指数项相互竞争，产生带有 $\langle S \rangle \gg 1$ 的弱耦合极小值。

The second idea was to invoke T -duality of the EFT, which for a single complex field T is the $SL(2, \mathbb{Z})$ action on T [254-256], with S being invariant. Imposing that the Kähler-invariant combination $K + \log |W|^2$ is invariant then implies that the non-perturbative superpotential should transform as a modular form of weight 3 [255, 256]:

第二个思路是利用有效场论的 T 对偶性，对于单个复场 T，它是作用在 T 上的 $SL(2, \mathbb{Z})$ 变换 [254-256]，其中 S 保持不变。要求凯勒不变组合 $K + \log |W|^2$ 不变即可推得，非微扰超势应当变换为权 3 的模形式 [255, 256]:

$$W(S,T) = \frac{F(j(T), S)}{\eta(T)^6},\ W \to (icT+d)^{-3} W,\ T \to \frac{aT - ib}{icT + d},$$
$$ad - bc = 1, \tag{160}$$

---

[42] Fluxes and non-perturbative effects were combined with α' corrections to the Kähler potential to find a version of LVS in the heterotic string in [250].

[42] 通量与非微扰效应结合，再加上凯勒势的 α' 修正，于文献 [250] 中得到了杂弦中的大体积紧致化 (LVS) 版本。

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where $\eta(T)$ is the Dedekind η function and $j(T)$ is the absolute modular invariant function. Concrete calculations of the one-loop threshold corrections to the holomorphic gauge kinetic function give $f = S + \alpha \log \eta(T)$ [257]. This leads to moduli stabilization of the T field at $\langle T \rangle = 1.2$ [255] in string units (see also [256]), with a negative cosmological constant, also of string scale, and with broken supersymmetry.

其中 $\eta(T)$ 是戴德金 η 函数，$j(T)$ 是绝对模不变函数。对全纯规范动力学函数的单圈阈值修正的具体计算给出了 $f = S + \alpha \log \eta(T)$ [257]。这使得 T 场在弦单位下的 $\langle T \rangle = 1.2$ 处实现模稳定 [255](也见文献 [256])，此时宇宙学常数为负，大小为弦标度，且超对称被破缺。

Generalizations to functions $F(j(T), S) = \Omega(S) G_4^m G_6^n P(j) / \eta^{8m+12n}$, where G_4, G_6 are the holomorphic Eisenstein functions of weight 4 and 6 respectively, and $P(j(T)$ is a polynomial of $j(T)$, were studied in [258], and found to generate potentials with perturbatively stable AdS vacua. Most of these configurations have minima at the fixed points $T = 1$ and $T = e^{i\pi/6}$, which are always extrema and automatically preserve supersymmetry, but there exist other configurations in which supersymmetry is broken [258]. Recent generalizations to include concrete toroidal orbifold models [259] and potential de Sitter minima have been studied in [260, 261].

文献 [258] 研究了推广到函数 $F(j(T), S) = \Omega(S) G_4^m G_6^n P(j) / \eta^{8m+12n}$ 的情形: 其中 G_4, G_6 分别是权为 4 和 6 的全纯爱森斯坦函数，$P(j(T)$ 是 $j(T)$ 的多项式，研究发现这类推广能生成微扰下稳定的反德西特真空的势。大多数这类构型的极小值出现在不动点 $T = 1$ 和 $T = e^{i\pi/6}$ ，这些位置始终是极值且自动保持超对称，但也存在其他超对称破缺的构型 [258]。近期的推广工作已经纳入了具体的环面轨形模型 [259]，且文献 [260, 261] 研究了可能的德西特极小值。

In Calabi-Yau compactifications of the heterotic string, H_3 flux can at most fix the complex structure moduli. This limitation has motivated studying the heterotic string compactified on more general spaces, including $SU(3)$ structure and half-flat manifolds [262-264]. Furthermore, in addition to the geometric moduli, heterotic compactifications have vector bundle moduli (see, e.g., [265]) which are singlets of the corresponding gauge group for $(0, 2)$ or nonstandard-embedding compactifications. Stabilizing them is also a challenge, similar to that of open string moduli in type II strings. A difference with type II models is the fact that in heterotic models the gauge sector is in the bulk, and so there is no route to a modular approach that separates the task of moduli stabilization from that of constructing the Standard Model. In the heterotic string, the two challenges need to be addressed simultaneously.

在杂弦的卡拉比-丘紧致化中，H_3 通量最多只能固定复结构模。这一局限推动了研究者对更一般空间上杂弦紧致化的研究，包括 $SU(3)$ 结构流形和半平流形 [262-264]。此外，除几何模外，杂弦紧致化还存在向量丛模 (例如参见 [265])，对于 $(0, 2)$ 或非标准嵌入紧致化，这些模是对应规范群的单态。固定它们也是一项挑战，类似于 II 型弦中的开弦模问题。杂弦模型与 II 型模型的一个区别是，杂弦模型的规范扇区在体空间中，因此不存在将模固定任务与构造标准模型任务分离的模块化方法。在杂弦中，这两项挑战必须同时解决。

A third approach [266] began by revisiting the apparent obstacle to using (157) to stabilize at weak coupling, which is that for an integrally quantized three-form $dB \in H^3(X, \mathbb{Z})$, the superpotential term $\int \mathrm{d}B \wedge \Omega$ cannot be small. However, in the heterotic string, the three-form flux is given by

第三种方法 [266] 首先重新审视了在弱耦合下使用 (157) 进行固定的明显障碍: 对于整数量子化的三形式 $dB \in H^3(X, \mathbb{Z})$ ，超势项 $\int \mathrm{d}B \wedge \Omega$ 不可能很小。但在杂弦中，三形式通量由下式给出

$$H = \mathrm{d}B + \frac{\alpha'}{4}\Omega_3(A) - \frac{\alpha'}{4}\Omega_3(\omega), \tag{161}$$

where for a gauge connection A we define the Chern-Simons three-form $\Omega_3(A)$ by

其中对于规范联络 A ，我们将陈-西蒙斯三形式 $\Omega_3(A)$ 定义为

$$\Omega_3(A) \;:=\; A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A. \tag{162}$$

Similarly, $\Omega_3(\omega)$ is the Chern-Simons three-form built from the spin connection ω . Integrating over a three-cycle $Q \in H_3(X, \mathbb{Z})$, we define the Chern-Simons invariant:

类似地，$\Omega_3(\omega)$ 是由自旋联络 ω 构造的陈-西蒙斯三形式。对三循环 $Q \in H_3(X, \mathbb{Z})$ 积分后，我们定义陈-西蒙斯不变量:

$$\mathrm{CS}(A, Q) \;:=\; \int_Q \Omega_3(A). \tag{163}$$

In general, $\mathrm{CS}(A, Q)$ is a rational number, not necessarily an integer. The proposal of [266] was to consider X_6 with $Q \subset X_6$ supporting fractional Chern-Simons invariants, take $\mathrm{d}B = 0$, and embed the spin connection in the gauge connection, in such a way that the total flux superpotential is small. Writing $\check{\Omega}$ for the three-cycle Poincaré dual to Ω , we have, for $\mathrm{d}B = \Omega_3(\omega) = 0$:

一般来说，$\mathrm{CS}(A, Q)$ 是有理数，不一定是整数。文献 [266] 的方案是考虑带有支撑分数陈-西蒙斯不变量的 $Q \subset X_6$ 的 X_6 ，取 $\mathrm{d}B = 0$ ，并将自旋联络嵌入规范联络，使得总通量超势很小。将 $\check{\Omega}$ 记为 Ω 的庞加莱对偶三循环，对于 $\mathrm{d}B = \Omega_3(\omega) = 0$ 我们有:

$$\int H \wedge \Omega = -\frac{\alpha'}{4} \int_{\check{\Omega}} \Omega_3(A) = -\frac{\alpha'}{4}\, \mathrm{CS}\left(A, \check{\Omega}\right), \tag{164}$$

and the object on the right is not restricted by integer quantization.

且右侧的对象不受整数量子化的约束。

Examples were given in [266] in which A is a connection on a flat bundle with nontrivial holonomies, i.e., where A is characterized by its Wilson lines, and $\int H \wedge \Omega$ is smaller than integral quantization would have required. Stabilization of all moduli was studied in [250], while $\mathrm{CS}(A, Q)$ was very carefully computed in a number of explicit examples, some with flat connections (Wilson lines) and others with curvature, in [267, 268]. These constructions illustrate the inevitable connection between the visible sector and moduli stabilization in the heterotic string: some of the choices of visible sector Wilson lines proposed in the literature to break the visible E_8 to the Standard Model actually introduce nonvanishing fractional Chern-Simons invariants, and so break supersymmetry via (164) [266].

文献 [266] 给出了相关例子，其中 A 是带非平凡和乐的平坦丛上的联络，即 A 由其威尔逊线表征，且 $\int H \wedge \Omega$ 比整数量子化要求的更小。文献 [250] 研究了所有模的固定，而文献 [267, 268] 在多个明确例子中仔细计算了 $\mathrm{CS}(A, Q)$ ，部分例子采用平坦联络 (威尔逊线)，部分例子带有曲率。这些构造说明了杂弦中可见部分与模固定之间存在必然联系: 文献中提出的、用于将可见 E_8 破缺到标准模型的部分可见部分威尔逊线选择，实际上会引入非零的分数陈-西蒙斯不变量，因此会通过 (164) 破缺超对称 [266]。

Type I

I 型弦论

Even though type I theories can be thought of as orientifolded versions of type II, there are mechanisms that are most readily developed in the type I context. Fluxes of three forms combined with magnetic fluxes give a rich range of possibilities for moduli stabilization. Concrete models in terms of magnetized D9-branes with toroidal orbifold compactifications have been studied in [269,270]; see also [271].

尽管 I 型理论可视为 II 型弦论的定向投影版本，但 I 型框架下存在许多更易于研究的机制。三形式通量结合磁通量为模稳定提供了丰富的可能性。针对带磁 D9 膜的环面轨形紧致化具体模型已在文献 [269,270] 中得到研究；另参见文献 [271]。

M-Theory

M 理论

Four-dimensional $\mathcal{N} = 1$ vacua result from compactifications of 11-dimensional supergravity, which is the low-energy limit of M-theory, on 7-manifolds of G_2 holonomy [272].

四维 $\mathcal{N} = 1$ 真空来源于 M 理论低能极限下的 11 维超引力在 G_2 和乐 7 流形上的紧致化 [272]。

Suppose that X_7 is a compact manifold of G_2 holonomy, with $N = b_3(X_7)$, and let $\{\sum_i\}, i = 1, \ldots, N$ be a basis of independent three cycles. Defining $\ell_M^9 = 4\pi\kappa_{11}^2$ and introducing the G_2 -invariant three-form Φ , we can write the geometric moduli as [273]

设 X_7 是一个 G_2 和乐紧致流形，满足 $N = b_3(X_7)$ ，令 $\{\sum_i\}, i = 1, \ldots, N$ 为独立 3-循环的一组基。定义 $\ell_M^9 = 4\pi\kappa_{11}^2$ 并引入 G_2 不变三形式 Φ ，我们可将几何模写为 [273]

$$z_i \equiv t_i + i s_i := \int_{\Sigma_i} C_3 + i \int_{\Sigma_i} \Phi. \tag{165}$$

Then, the volume $\mathcal{V}_7 = \mathrm{Vol}(X_7)/\ell_M^7$ is a homogeneous function of the s_i , of degree 7/3 [274], and the Kähler potential is

此时，体积 $\mathcal{V}_7 = \mathrm{Vol}(X_7)/\ell_M^7$ 是 s_i 的齐次函数，次数为 7/3[274]，凯莱势能为

$$K = -3\ln\left(4\pi^{1/3}\mathcal{V}_7(s_1, \ldots s_N)\right). \tag{166}$$

The G_4 flux of M-theory gives rise to a classical superpotential [274,275] - for statistics of the resulting flux vacua, see [273]. Additional non-perturbative contributions to the superpotential arise from gaugino condensation in gauge sectors supported on singularities in X_7 , as well as from Euclidean M2-branes wrapping three cycles calibrated by Φ (see, e.g.,[276]).

M 理论的 G_4 通量会产生经典超势 [274,275]——所得通量真空的统计性质参见 [273]。超势还存在额外非微扰贡献，来源于 X_7 奇点上支撑的规范 sector 中的戈迪诺凝聚，以及包裹 Φ 标定三循环的欧几里得 M2 膜 (参见例如 [276])。

The phenomenology of G_2 compactifications was explored in [277,278], in a class of scenarios in which the gaugino condensate superpotential alone, with vanishing fluxes, was argued to stabilize all moduli.

G_2 紧致化的唯象学研究可见 [277,278]，该文中讨论了一类仅依靠戈迪诺凝聚超势 (通量为零) 即可稳定所有模的方案。

Overall, despite much study of the geometry and physics of manifolds of G_2 holonomy (see, e.g., [279-284] and references therein), the details of moduli stabilization are not yet as fully developed, especially in the more realistic setups with singularities, as in the string theory case.

整体而言, 尽管已有大量针对 G_2 和乐流形几何与物理的研究 (参见例如 [279-284] 及其中参考文献), 模稳定的细节仍未得到充分发展，尤其是带奇点的更真实 setup 中，远不及弦理论情形发展完善。

General Mechanisms

一般机制

We have now seen many proposals for moduli stabilization from different corners of the string/M-theory moduli space, some of them with the potential to give rise to de Sitter solutions. Even though the implementations differ, the core structures are similar, and the strategy is always to incorporate a sufficiently rich collection of sources that are present in string theory.

我们现在已经看到弦/M 理论模空间不同领域提出的诸多模稳定方案，其中部分方案有望得到德西特解。尽管具体实现方式各不相同，但核心结构是相似的，研究思路始终是引入弦理论中存在的足够丰富的源。

We have extensively discussed the role of fluxes, D-branes, anti-D-branes, orientifolds, and quantum effects. Let us now consider a more complete list of possible sources: following the discussion in [285], we classify the various sources of nonderivative terms in the four-dimensional action from the higher-dimensional theory, without assuming that the starting point is supersymmetric or that the compactification is Ricci-flat.

我们已经详细讨论过通量、D 膜、反 D 膜、定向亏平面以及量子效应的作用。下面我们来列出更完整的可能源: 遵循文献 [285] 中的讨论，我们对高维理论在四维作用量中产生的各类非导数项源进行分类，不假设出发点是超对称的，也不假设紧化是里奇平坦的。

- Noncritical strings. Strings in dimensions above the critical dimension provide a positive contribution to the vacuum energy [285]:

- 非临界弦。临界维数以上的弦对真空能贡献正的项 [285]:

$$V_{\mathrm{nc}} \propto \frac{(D - D_{\mathrm{crit}})\, e^{2\phi}}{\mathcal{V}}, \tag{167}$$

and can give rise to anti-de Sitter and de Sitter vacua in four dimensions [286, 287]; see [212] for a recent critical assessment of this mechanism.

并且可以在四维中生成反德西特和德西特真空 [286, 287]; 关于该机制的最新批判性评估参见 [212]。

- Curvature of compact space. In the preceding sections, we largely restricted to compactifications admitting Ricci-flat metrics, so that the contribution of the internal space to the Einstein-Hilbert action vanishes. In a more general compactification on a compact six-manifold X_6 with curvature $\mathcal{R}_6$, one has [285]:

- 紧致空间曲率。在前文章节中，我们基本局限于允许里奇平坦度规的紧化，因此内部空间对爱因斯坦-希尔伯特作用量的贡献为零。在更一般的情况下，对曲率为 $\mathcal{R}_6$ 的紧致六流形 X_6 做紧化，可得 [285]:

$$V_{\mathrm{curv}} \propto -\frac{e^{2\phi}}{\mathcal{V}^2}\int_{X_6} \mathcal{R}_6 \tag{168}$$

Thus, the curvature contribution to the four-dimensional potential is negative if X_6 is a positively curved space, such as a sphere, and is positive if X_6 is a negatively curved space, such a product of Riemann surfaces of genus $g > 1$. Hyperbolic compactifications have been explored as a means of obtaining de Sitter solutions from 11 dimensions in [288] (for previous work in this direction, see [289]). An important lesson from the two-dimensional case is that negatively curved manifolds are much more numerous than positively curved manifolds.

因此，若 X_6 是球面这类正曲率空间，曲率对四维势的贡献为负；若 X_6 是亏格为 $g > 1$ 的黎曼曲面乘积这类负曲率空间，贡献则为正。文献 [288] 已经研究了双曲紧化，将其作为从 11 维得到德西特解的方法 (该方向的早期研究参见 [289])。二维情形给出的一个重要结论是: 负曲率流形的数量远多于正曲率流形。

- Fluxes. As we have seen, fluxes of antisymmetric tensor fields make positive contributions to the scalar potential. The positivity of the flux contribution is a direct consequence of the positivity of the corresponding kinetic terms in the higher-dimensional theory.

- 通量。正如我们所见，反对称张量场的通量对标量势贡献正的项。通量贡献为正，是高维理论中对应动能项为正的直接结果。

- Branes. In principle branes contribute positively to the scalar potential, but due to their BPS nature, this contribution often cancels against other sources in a full solution. Antibranes, as well as any non-BPS branes, contribute positively to the scalar potential.

- 膜。原则上膜对标量势贡献正的项，但由于其 BPS 性质，在完整解中该贡献常常会与其他源抵消。反膜以及所有非 BPS 膜都会对标量势贡献正的项。

- Orientifolds. Orientifolds contribute a similar amount as D-branes, but with a negative sign if they have negative tension.

- 定向亏平面。定向亏平面的贡献量与 D 膜相近，但如果它具有负张力，贡献符号为负。

- Perturbative corrections. We have seen that such corrections contribute with different powers of e^{ϕ} and $1/\mathcal{V}$, and the sign can be positive or negative.

- 微扰修正。我们已经看到，这类修正以 e^{ϕ} 和 $1/\mathcal{V}$ 的不同幂次贡献，符号可正可负。

- Non-perturbative effects. Non-perturbative effects contribute terms of order $e^{-c\rho}$ where ρ is the modulus (Kähler or dilaton) measuring the gauge coupling. Again either sign is possible.

- 非微扰效应。非微扰效应贡献量级为 $e^{-c\rho}$ 的项，其中 ρ 是衡量规范耦合的模 (凯勒模或胀子)，符号同样可正可负。

It is important to emphasize that the effects enumerated above exist, and it is not justified to ignore them. Combining some or all of these effects can naturally give rise to vacua for the moduli fields at the level of the EFT analysis.

需要着重强调的是，上述列举的效应都是实际存在的，忽略它们是不合理的。在有效场论 (EFT) 分析层面，结合其中部分或全部效应可以自然得到模场的真空。

However, the devil is in the details: finding minima in the regime of validity of the EFT is highly non-trivial. There is an enormous space of possible models, very little of which has been populated to date with full-fledged and explicit constructions. The cause of this state of affairs is not that no such constructions can exist, but just that quantum gravity is relatively difficult, and writing down a mathematical model of a realistic universe takes a certain amount of work. Moreover, although the concrete realizations produced to date may look baroque, the general principles are clear and clean.

但难点在于细节: 在 EFT 的有效适用范围内找到极小值绝非易事。可能的模型空间极其广阔，迄今为止只有极少部分得到了全面明确的构造。出现这种情况不是因为这类构造不可能存在，只是量子引力本身难度较高，构建一个符合真实宇宙的数学模型需要相当多的工作。此外，尽管目前已有的具体构造看起来繁杂粗糙，但一般原理清晰明确。

Cosmology

宇宙学

Arguably the most important application of moduli stabilization is to the study of cosmology in string theory. The dark universe involves three main unknowns: dark matter and dark energy today and the inflationary energy at very early times. Moduli are natural candidates for all three unknowns. Moreover, the dynamics of moduli fields encodes the evolution of the extra dimensions of string theory, and can impact early stages of cosmic history.

可以说，模稳定最重要的应用就是弦理论中的宇宙学研究。暗宇宙包含三大未知: 当今的暗物质、暗能量，以及极早期宇宙的暴胀能量。模正是这三类未知的自然候选者。此外，模场的动力学包含了弦理论额外维度的演化，还能影响宇宙史的早期阶段。

Inflation, a transient period of accelerated expansion, is the leading scenario to describe early-universe cosmology. Inflationary theory has impressive achievements, including solving the horizon and flatness problems of the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology and, more importantly, explaining the observed pattern of density perturbations imprinted in the cosmic microwave background (CMB). However, inflation itself does not have an explanation: it is a framework in search of a theory. String theory may be that theory.

暴胀是一段短暂的加速膨胀时期，是描述早期宇宙学的主流模型。暴胀理论拥有诸多亮眼成果，包括解决了弗里德曼-勒梅特-罗伯逊-沃尔克 (FLRW) 宇宙学的视界问题与平坦性问题， 更重要的是，它解释了宇宙微波背景 (CMB) 中观测到的密度扰动模式。但暴胀本身仍缺乏根源性解释: 它是一个有待找到底层理论的框架，弦理论或许就是这个理论。

Having a period of accelerated expansion tends to dilute whatever physics existed at high energies, so one could imagine that string-theoretical signatures might be hidden from potential observations. Fortunately, the moduli tend to be light enough to survive after inflation, and may play a relevant role after inflation: not only in the early universe, between inflation and nucleosynthesis, but also at late times in the form of dark energy. This motivates the study of cosmology within moduli stabilization [43] .

一段加速膨胀期往往会稀释所有高能阶段存在的物理过程, 因此人们可能会认为弦理论的特征很难被潜在观测捕捉到。幸运的是，模通常足够轻，能够在暴胀后留存下来，并可能在暴胀后发挥重要作用: 不仅在暴胀与核合成之间的早期宇宙中，还能以暗能量的形式存在于晚期宇宙中。这推动了模稳定框架下弦论宇宙学的研究 [43] 。

Inflation

暴胀

Inflation was historically understood as a mechanism, and a phenomenon, in quantum field theory coupled to general relativity. If one temporarily and counterfactually disregards all effects of the quantization of gravity, inflation is a striking success: its predictive and explanatory power far exceeds the new questions that it raises. In particular, inflation predicted that the universe should be spatially flat to good approximation, and it predicted that the CMB should have fluctuations that are nearly, but not exactly, scale-invariant; nearly Gaussian; and correlated on superhorizon scales. All these predictions were decisively confirmed at the start of the era of precision cosmology.

从历史发展来看，暴胀是耦合广义相对论的量子场论中的一种机制与现象。如果暂时忽略引力量子化的所有效应 (这并非实际情况)， 暴胀是一项惊人的成就: 它的预言与解释能力远胜于它提出的新问题。具体而言，暴胀预言宇宙在空间上应当非常接近平直，还预言宇宙微波背景 (CMB) 的涨落几乎 (但不完全) 是标度不变的，近似高斯分布，且在超视界尺度上存在关联。所有这些预言都在精度宇宙学时代伊始得到了确凿证实。

Inflation led to new questions: what is the inflaton field? What dynamics led to the initial conditions for inflation? String theory has the potential to address these questions: in particular, it is replete with candidates for the inflaton field, in the form of the many axions and moduli found in typical compactifications. At the same time, string theory sharpens one of the central problems of inflationary cosmology: why is the inflaton potential flat enough to support prolonged accelerated expansion?

> 暴胀也带来了新问题: 暴胀子场是什么？是什么动力学过程产生了暴胀的初始条件？弦理论有潜力解决这些问题: 具体来说，弦理论中存在大量暴胀子场的候选者，就是典型紧致化中常见的众多轴子与模。同时，弦理论也放大了暴胀宇宙学的一个核心问题: 为什么暴胀子势足够平坦，能支撑长时间的加速膨胀?

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[43] String theory may have cosmological signatures beyond inflation (see for instance the recent review [7] and references therein). However, the implementation of alternatives to inflation is challenging: EFT descriptions are often insufficient, and the correspondence with observations is less developed than in the case of inflation. In this sense inflation appears to be preferred both by experimental evidence and by theoretical considerations. Even so, it is prudent to explore and develop alternatives.

> [43] 弦理论中除暴胀外还可能存在其他宇宙学信号 (例如近期综述 [7] 及其中的参考文献)。然而，实现暴胀的替代方案颇具挑战: 有效场论 (EFT) 描述通常不够充分，和观测的对应关系也不如暴胀理论完善。从这个角度看，无论是实验证据还是理论考量，暴胀都更受青睐。即便如此，探索和发展替代方案仍是谨慎之举。

To see this, consider a single real scalar field ϕ , a candidate inflaton, coupled to general relativity, whose Lagrangian density at tree level is

> 举个例子，考虑一个候选暴胀子——单个实标量场 ϕ ，它耦合广义相对论，其树层级拉格朗日密度为

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V_0(\phi)\,, \tag{169}$$

where $V_0(\phi)$ is the tree-level (i.e., classical) scalar potential. Including terms with at most two derivatives, the quantum-corrected Lagrangian density - incorporating the effects of loops of the light fields, which are the inflaton and the graviton - takes the form:

> 其中 $V_0(\phi)$ 是树层级 (即经典) 标量势。考虑最多含二阶导数的项后，纳入轻场 (暴胀子和引力子) 的圈效应修正后的量子拉格朗日密度形式为:

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\mathcal{R} - \frac{1}{2}f(\phi)(\partial\phi)^2 - V(\phi)\,, \tag{170}$$

where $f(\phi)$ and $V(\phi)$ are two functions of ϕ , and we have used the freedom to rescale to Einstein frame to absorb a possible additional function of ϕ from the Einstein-Hilbert term. The effect of $f(\phi)$ is generally small (see [6] for a detailed discussion), and so we will set $f(\phi) \to 1$ henceforth.

其中 $f(\phi)$ 和 $V(\phi)$ 是 ϕ 的两个函数，我们利用重标度自由度转到爱因斯坦框架，吸收了爱因斯坦-希尔伯特项中可能存在的额外 ϕ 函数。$f(\phi)$ 的效应通常很小 (详细讨论见 [6])，因此我们接下来将令 $f(\phi) \to 1$ 。

A sufficient condition for the theory (170) to support a prolonged period of inflation is that the slope and curvature of the potential are small compared to the Planck mass: given suitable initial conditions, the potential energy can then dominate over the kinetic energy, with $p \approx -\rho$, and can do so for many e -folds of expansion. Specifically, slow-roll inflation is possible if

理论 (170) 支撑长时间暴胀的一个充分条件是，势的斜率和曲率与普朗克质量相比很小: 只要初始条件合适，势能就会主导动能，满足 $p \approx -\rho$ ，并且这种状态可以维持很多次 e 倍膨胀。具体来说，慢滚暴胀满足以下条件就可以存在:

$$\varepsilon := \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \quad \text{and} \quad \eta := M_{\rm pl}^2 \frac{V''}{V} \ll 1, \tag{171}$$

with primes denoting derivatives $\frac{d}{d\phi}$.

撇号表示对 $\frac{d}{d\phi}$ 求导。

In general there is no systematic relation between $V_0(\phi)$ and $V(\phi)$, so having the classical potential $V_0(\phi)$ fulfill the slow-roll conditions (171) does not imply that the full potential $V(\phi)$ will do so. In other words, quantum corrections to the potential can affect whether inflation is possible.

一般来说，$V_0(\phi)$ 和 $V(\phi)$ 之间不存在固定关系，因此经典势 $V_0(\phi)$ 满足慢滚条件 (171) 不代表完整势 $V(\phi)$ 也满足。换句话说，势的量子修正会影响暴胀是否能发生。

In the important case that $V_0(\phi)$ preserves an approximate shift symmetry, so will $V(\phi)$. In particular, if

当 $V_0(\phi)$ 保持近似平移对称性时，$V(\phi)$ 也会保持该对称性，这是非常重要的情况。具体来说，如果

$$V_0(\phi) = \frac{1}{2} m_0^2 \phi^2, \tag{172}$$

with $m_0 \ll M_{\rm pl}$, then one finds:

满足 $m_0 \ll M_{\rm pl}$ ，就可以得到:

$$V(\phi) = V_0(\phi) + \mathcal{O}\left(V_0/M_{\rm pl}^4\right) \approx V_0(\phi). \tag{173}$$

Thus, a small inflaton mass is radiatively stable: its smallness is not spoiled by loops of the light fields.

因此，很小的暴胀子质量在辐射修正下是稳定的: 它的小质量不会被轻场圈修正破坏。

An argument that was historically influential, but leads to erroneous conclusions, states that the correction term $\mathcal{O}\left(V_0/M_{\rm pl}^4\right)$ in (173) captures the leading quantum gravity corrections to the effective theory (169), and so inflation with the potential $\frac{1}{2}m_0^2\phi^2$ is not impacted by the quantization of gravity. The logical flaw is the following: if (169) were the classical theory for ϕ and the massless graviton, obtained after integrating out all the massive degrees of freedom of quantum gravity, then indeed the remaining corrections from loops of the light fields would be negligible. But deep principles of quantum gravity appear to forbid exact shift symmetries [44], such as would occur with $m_0 \to 0$, and so the question of how small m_0 can be is already a quantum gravity question: it can only be resolved with knowledge of the ultraviolet completion of gravity.

有一个在历史上颇具影响力但结论错误的观点认为，式 (173) 中的修正项 $\mathcal{O}\left(V_0/M_{\mathrm{pl}}^4\right)$ 包含了有效理论 (169) 中领头阶量子引力修正，因此势能为 $\frac{1}{2}m_0^2\phi^2$ 的暴胀不受引力量子化的影响。其逻辑缺陷如下: 如果式 (169) 是 ϕ 和无质量引力子的经典理论，是积分掉量子引力所有有质量自由度后得到的，那么轻场圈带来的剩余修正确实可以忽略。但量子引力的深层原理似乎禁止了精确平移对称性 [44] (当存在 $m_0 \to 0$ 时就会出现这种对称性)，因此 m_0 能有多小本身就是一个量子引力问题: 只有知晓引力的紫外完备性才能解答。

The terminology is one cause of the confusion. The Wilsonian EFT obtained by integrating out heavy fields, and high-momentum modes of light fields, is reasonably termed the classical theory of the light fields, in which one can still compute quantum corrections from loops of the light fields. However, the nature and couplings of the heavy fields, which in turn impact the form of the classical EFT, are dictated by the ultraviolet completion, i.e., by the quantum theory of gravity.

术语是造成混淆的原因之一。通过积分掉重场和轻场的高动量模式得到的威尔逊有效场论，可以被合理地称为轻场的经典理论，在该理论中仍可以计算轻场圈的量子修正。然而，重场的性质与耦合 (进而影响经典有效场论的形式) 是由紫外完备性，即量子引力理论决定的。

To further illustrate this point, consider two candidate theories of quantum gravity, $\mathcal{T}_1$ and $\mathcal{T}_2$, each with some spectrum of massive states with $M > M_{\mathrm{pl}}$. Suppose that at low energies $\mathcal{T}_1$ contains a light field ϕ with an accidentally flat potential $V_{\mathcal{T}_1}$, such that $\varepsilon\left(V_{\mathcal{T}_1}\right), \eta\left(V_{\mathcal{T}_1}\right) \ll 1$: then slow-roll inflation is possible in the theory $\mathcal{T}_1$. If the spectra of heavy states in $\mathcal{T}_1$ and $\mathcal{T}_2$ differ only by factors of order unity, one would be tempted to call $\mathcal{T}_1$ and $\mathcal{T}_2$ "similar" ultraviolet completions of gravity, and to claim that their low-energy phenomenology should be similar as well. However, the slow-roll parameter η is highly sensitive to the Planck-scale spectrum, and one generally finds $\eta\left(V_{\mathcal{T}_2}\right) \approx 1$, so that inflation does not occur in the theory $\mathcal{T}_2$. Thus, whether inflation occurs or not can be changed by very subtle changes to the spectrum of states at masses $M > M_{\mathrm{pl}}$.

为进一步说明这一点，我们考虑两种候选量子引力理论 $\mathcal{T}_1$ 和 $\mathcal{T}_2$，二者都存在质量为 $M > M_{\mathrm{pl}}$ 的有质量能谱。假设在低能下，$\mathcal{T}_1$ 包含一个轻场 ϕ，其具有偶然平坦势 $V_{\mathcal{T}_1}$，满足 $\varepsilon\left(V_{\mathcal{T}_1}\right), \eta\left(V_{\mathcal{T}_1}\right) \ll 1$: 那么理论 $\mathcal{T}_1$ 中就可以发生慢滚暴胀。如果 $\mathcal{T}_1$ 和 $\mathcal{T}_2$ 的重场能谱仅相差量级为 1 的因子，人们很容易会将 $\mathcal{T}_1$ 和 $\mathcal{T}_2$ 称为“相似”的引力紫外完备，并声称它们的低能唯象也应当相似。然而，慢滚参数 η 对普朗克能标下的能谱高度敏感，通常会得到 $\eta\left(V_{\mathcal{T}_2}\right) \approx 1$，因此理论 $\mathcal{T}_2$ 中不会发生暴胀。由此可见，质量为 $M > M_{\mathrm{pl}}$ 的能谱发生极细微的改变，就会改变暴胀能否发生的结论。

In summary, quantum gravity effects do not decouple from inflation, and the η parameter is sensitive to the ultraviolet completion. This fact is termed the eta problem. In some older papers, this issue is called the supergravity eta problem, but it is in no way special to supergravity. Indeed, the eta problem is nothing more than the statement that an inflaton mass that is small compared to the cutoff scale is unnatural in effective field theory.

总而言之，量子引力效应不会退耦于暴胀，且 η 参数对紫外完备性十分敏感。这一事实被称为 η 问题。在一些较早的论文中，这个问题被称作超引力 η 问题，但它绝非超引力所特有。实际上，η 问题不过是这样一个结论: 在有效场论中，暴胀子质量小于截断能标是不自然的。

Another statement of the problem is as follows: if the low-energy potential is

该问题的另一种表述如下: 如果低能势为

$$V(\phi) = V_0(\phi)\left(1 + \sum_{\delta} c_{\delta}\left(\frac{\phi}{M_{\rm pl}}\right)^{\delta}\right) \tag{174}$$

with Wilson coefficients c_{δ} , then in the absence of any additional structure, one needs to know the c_{δ} with $\delta \lesssim 2$ to at least $\mathcal{O}(1\%)$ accuracy in order to exhibit slow-roll inflation. That is, slow-roll inflation is affected by Planck-suppressed operators up to dimension $\Delta = \delta + 4 \approx 6$.

且威尔逊系数为 c_{δ} ，那么在不存在任何额外结构的情况下，要实现慢滚暴胀，需要将 c_{δ} 的 $\delta \lesssim 2$ 精度控制到至少达到 $\mathcal{O}(1\%)$ 。也就是说，慢滚暴胀会受到最高到 $\Delta = \delta + 4 \approx 6$ 维的普朗克压低算符影响。

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[44] Some of the sharpest arguments against exact global symmetries in quantum gravity were developed in the context of the weak gravity conjecture [290]; see the reviews [291-294].

[44] 针对量子引力中精确整体对称性的一些最尖锐的论证，是在弱引力猜想的框架下发展出来的 [290]；综述参见文献 [291-294]。

This state of affairs is an extraordinary opportunity for string theory. Knowledge of the structure of quantum gravity is essential in order to interpret the results of precision CMB observations made over the past two decades.

这种现状对弦理论来说是绝佳的机遇。要解读过去二十年间精密宇宙微波背景观测的结果，掌握量子引力的结构知识必不可少。

Inflation in String Theory

弦论中的暴胀

The arguments just presented are general facts about a low-energy effective field theory of quantum gravity, and did not rely on details of string theory. However, having a concrete ultraviolet completion in hand dramatically sharpens the picture [45] .

刚刚给出的论证是量子引力低能有效场论的通用结论，并不依赖弦论的细节。然而，拥有一个具体的紫外完备化会大幅让图像变得清晰 [45] 。

Early works on inflation in string theory, with closed string moduli serving as the inflaton, include [295, 296]. The discovery of D-branes led to the idea of brane inflation [297], developed in detail in [298] (see also [299, 300]), in which the inflaton is the separation of a brane-antibrane pair. The end of inflation is triggered by condensation of the open string tachyon, potentially leaving cosmic superstrings as relics [301]. However, these studies appeared before the emergence of concrete scenarios for the stabilization of all moduli, and so the treatment of moduli couplings was necessarily incomplete.

弦论暴胀的早期工作以闭弦模作为暴胀子，包括文献 [295, 296]。D 膜的发现催生了膜暴胀的想法 [297]，该想法在 [298] 中得到了详细发展 (另见 [299, 300])，在这一模型中，暴胀子是 D 膜-反 D 膜对的间距。暴胀的结束由开弦快子凝聚触发，有可能留下宇宙超弦遗迹 [301]。但这些研究都完成于所有模稳定的具体方案出现之前，因此对模耦合的处理必然是不完备的。

The first analysis of inflation in a compactification with stabilized moduli appeared in [143], which considered a D3-brane moving toward an anti-D3-brane in a Klebanov-Strassler throat region, in the context of a KKLT vacuum. The first key finding of [143] was that warping caused the Coulomb interaction of the D3-brane and anti-D3-brane to be extremely flat [46] . However, a more significant finding, with implications for a much wider range of models, was that moduli stabilization reintroduced the eta problem [143]. Specifically, stabilization of the Kähler moduli by non-perturbative effects introduced new inflaton mass terms, in such a way that the total inflaton potential could be flat enough for prolonged slow-roll inflation only if competing terms could be balanced against each other in a modest (percent-level) fine-tuning.

模稳定紧致化中首个暴胀分析出现在文献 [143]，该工作在 KKLT 真空的框架下，研究了向克莱班诺夫-斯特拉斯勒 throat 区域中反 D3 运动的 D3 膜。[143] 的第一个核心结论是，翘曲使得 D3 膜与反 D3 膜的库仑相互作用极为平缓 [46] 。但对更广范围的模型更具启示意义的一个更重要结论是，模稳定重新引入了 η 问题 [143]。具体而言，非微扰效应稳定凯勒模会给暴胀子引入新的质量项，只有当不同竞争项之间达到适度 (百分比量级) 的精细调谐平衡，总暴胀势才能足够平缓，维持长时间慢滚暴胀。

The principal model-dependent corrections to the inflaton mass in the scenario of [143] come from the inflaton dependence of Pfaffian factors in the non-perturbative superpotential (38). The effects on the inflaton potential of non-perturbative super-potential terms supported on D7-branes in the throat region were computed in [45] (building on [44]) and [303], while the general form of contributions from the bulk of the compactification was determined in [304] and [116]. The resulting phenomenology was analyzed in [305, 306]; for details see [6]. For a recent alternative implementation of brane inflation, see [78].

在 [143] 的方案中，暴胀子质量的主要模型依赖修正来自非微扰超势 (38) 中普法夫因子对暴胀子的依赖。throat 区域 D7 膜上非微扰超势项对暴胀势的影响在 [45](基于 [44]) 和 [303] 中完成了计算，而紧致化本体区域贡献的一般形式则由 [304] 和 [116] 确定。由此得到的唯象学在 [305, 306] 中得到分析；细节见文献 [6]。膜暴胀的近期替代实现可参见 [78]。

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[45] Comprehensive treatments of inflation in string theory appear in [6,7]; here we will limit ourselves to highlighting developments closely linked to moduli stabilization.

[45] 弦论暴胀的全面综述可见文献 [6,7]；本文我们仅重点介绍与模稳定紧密相关的研究进展。

[46] Warping also allowed for cosmic superstrings with tension far below existing limits [143,302].

[46] 翘曲还允许宇宙超弦的张力远低于现有观测极限 [143, 302]。

Closed string moduli are also promising inflaton candidates. In the context of LVS, Kähler moduli can drive inflation [130, 138]. In particular, for fibered Calabi-Yau compactifications with fiber modulus τ_f , the volume can be written as $\mathcal{V} \propto \tau_f F(\tau_i)+G(\tau_i)$ with $\tau_i \neq \tau_f, i=1,\ldots,h_{11}-1$ and F, G homogeneous functions of degree 1/2 and 3/2 , respectively. The fiber modulus τ_f can be an inflaton candidate: the scalar potential for the canonically normalized scalar field ϕ , defined as $\tau_f \propto e^{\alpha\phi}, \alpha = 1/\sqrt{3}$, is

闭弦模也是很有前景的暴胀子候选。在大体积弦论 (LVS) 框架下，凯勒模可以驱动暴胀 [130, 138]。特别地，对于带有纤维模 τ_f 的纤维化卡拉比-丘紧致化，体积可以写为 $\mathcal{V} \propto \tau_f F(\tau_i)+G(\tau_i)$ ，其中 $\tau_i \neq \tau_f, i=1,\ldots,h_{11}-1$ 和 F, G 分别是次数为 1/2 和 3/2 的齐次函数。纤维模 τ_f 可以成为暴胀子候选: 按如下方式定义的正则归一化标量场 ϕ 对应的标量势 $\tau_f \propto e^{\alpha\phi}, \alpha = 1/\sqrt{3}$ 为

$$V(\phi) = V_0\left(1 - ae^{-\alpha\phi} + \ldots\right), \tag{175}$$

where V_0 and a are computable constants. The ellipses include terms that are subdominant terms at large ϕ . Fibre™ inflation models lead to a potentially detectable primordial tensor signal, with a robust prediction for the spectral index $n_s = 0.969$, a tensor-to-scalar ratio $r = 0.007$, and a field displacement $\Delta\phi \simeq 5$ in Planck units. These models will be confronted by experiment in the next decade.

其中 V_0 和 a 是可计算的常数。省略号包含大 ϕ 下的次主导项。纤维暴胀模型可以产生潜在可探测的原初张量信号，对谱指数 $n_s = 0.969$ 、张标比 $r = 0.007$ 以及普朗克单位下的场位移 $\Delta\phi \simeq 5$ 都给出了稳健预言。这些模型将在未来十年接受实验检验。

The approximate flatness of the potential for ϕ is a consequence of classical scaling symmetries, as discussed in sections ”Scalings and Perturbative Expansions” and ”Calabi-Yau Compactifications”, for which ϕ can be seen as a pseudo-Goldstone boson [307, 308]. Even so, the Kähler moduli inflation models of [130, 138] are sensitive to the form of loop corrections to the Kähler potential. For a recent estimate of such corrections, see [211].

正如“标度与微扰展开”和“卡拉比-丘紧致化”小节所述，ϕ 势近似平坦是经典标度对称性的结果，在此框架下 ϕ 可被视为赝戈德斯通玻色子 [307, 308]。即便如此，文献 [130, 138] 的凯勒模胀子模型对凯勒势的圈修正形式十分敏感。关于这类修正的最新估算可参见文献 [211]。

If the inflaton is an axion, the axion shift symmetry can protect the potential against corrections [309]. Embedding the original natural inflation idea of [309] in string theory proved difficult because the requisite super-Planckian decay constants - of individual axions - have not been found in controlled parameter regimes [290, 310] [47] . Ideas for evading this problem include alignment of two or more axions [313] and collective displacement of many axions [314,315]. Axion inflation models in string theory and F-theory include [316,317].

若胀子是轴子，轴子的平移对称性可保护势不受修正 [309]。将文献 [309] 最初的自然暴涨思想嵌入弦论十分困难，因为在可控参数范围内尚未找到单个轴子所需的超普朗克衰变常数 [290, 310] [47] 。规避该问题的方案包括两个或多个轴子排列对齐 [313]，以及多个轴子集体位移 [314, 315]。弦论与 F 理论中的轴子暴涨模型可参见文献 [316, 317]。

Axion monodromy [318] achieves large-field inflation by winding through many periods of an axion with sub-Planckian periodicity. The axion monodromy model of [21] involves a type IIB flux compactification on a Calabi-Yau orientifold with $h^{1,1}_{-} > 0$: the inflaton is the two-form C_2 (see Table 1) on a cycle wrapped by an NS5-brane/anti-NS5-brane pair, and the inflaton potential is [21, 319]

轴子单环绕 [318] 通过让周期低于普朗克尺度的轴子缠绕多圈实现大场暴涨。文献 [21] 的轴子单环绕模型是包含 $h^{1,1}_{-} > 0$ 的卡拉比-丘定向对映上的 IIB 型通量紧致化: 胀子是 NS5 膜-反 NS5 膜对所缠绕周期上的二形式 C_2 (参见表 1)，暴涨势为 [21, 319]

$$V(\phi) = \mu^3 \phi + \Lambda^4 \cos\left(\frac{\phi}{f}\right), \tag{176}$$

where μ, f , and Λ are parameters with the dimensions of mass. The periodic modulation in (176) can give rise to oscillatory signatures in the CMB [319]. The mechanism of [21] is compatible with the KKLT scenario for moduli stabilization: the shift symmetry of C_2 protects the inflaton from some (but not all) potentially dangerous terms.

其中 μ, f 和 Λ 是质量量纲的参数。式 (176) 中的周期调制可在宇宙微波背景中产生振荡特征 [319]。文献 [21] 的机制与 KKLT 模稳定机制相容: C_2 的平移对称性可保护胀子不受部分 (而非全部) 潜在危险项的影响。

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[47] Strictly speaking, single axion periodicities somewhat greater than M_{pl} have been exhibited in the Lagrangians of consistent compactifications [311, 312], but the moduli potentials in these models do not allow the dynamics of large-field inflation: the axion energy in such a case would backreact disastrously.

[47] 严格来说，在自洽紧致化的拉格朗日中已经得到单个轴子周期略大于 M_{pl} 的结果 [311, 312]，但这些模型中的模势不支持大场暴涨动力学: 这种情况下轴子能量会产生灾难性的反作用。

The order parameter measuring the progress of axion monodromy inflation is the reduction of monodromy charge as the inflaton configuration unwinds. For example, in [21] the monodromy charge is D3-brane charge accumulated on the NS5-brane pair, while in [320] it is D3-brane charge induced on moving D7-branes. A very general problem in such models is that the stress-energy of monodromy charge backreacts on the internal space [21, 319, 321, 322] . As a result, one finds that some form of fine-tuning is required in all fully realized [48] axion monodromy scenarios, though the extent of the difficulty is model-dependent. To date, there is no fully explicit realization of axion monodromy inflation in a compact model, but there is also no known obstacle other than the complexity of the configuration: see, e.g., [323-330] for progress in this direction.

衡量轴子单环绕暴涨进程的序参量是胀子构型退绕时单环绕电荷的减少。例如,在文献 [21] 中单环绕电荷是积累在 NS5 膜对上的 D3 膜电荷, 而在文献 [320] 中, 单环绕电荷是运动 D7 膜上诱导的 D3 膜电荷。这类模型的一个普遍问题是单环绕电荷的应力能会对内部空间产生反作用 [21, 319, 321, 322]。因此，所有完全实现的 [48] 轴子单环绕方案都需要某种形式的精细调节，只是困难程度依模型而异。至今，紧致模型中尚无轴子单环绕暴涨的完全显式实现，但除了构型本身的复杂度外也不存在已知的障碍: 该方向的研究进展可参见例如文献 [323-330]。

After Inflation

暴涨后

One of the most important aspects of string moduli is that they naturally change the early history of the universe even after a period of inflation. Inflation tends to wash out physics at energies higher than the inflation scale, but moduli can survive in some circumstances.

弦模量最重要的特点之一是，即便在暴涨阶段结束后，它们也会自然改变宇宙的早期演化历史。暴涨通常会抹平暴涨能标以上的物理效应，但模量在某些情况下得以留存。

In supersymmetry-breaking compactifications, one can estimate the typical moduli masses to be $m_\phi \simeq m_{3/2}$, since the moduli couple gravitationally and are massless in the absence of supersymmetry breaking (in Minkowski space). In concrete cases this estimate is modified as follows:

在超对称破缺紧致化中，可以估计模量的典型质量为 $m_\phi \simeq m_{3/2}$ ，因为模量以引力强度耦合，且在闵氏空间中不发生超对称破缺时是无质量的。具体情况中这一估计会按如下方式修正:

$$\mathrm{KKLT}: \; m_\phi \simeq \ln\left(\frac{M_{\rm pl}}{m_{3/2}}\right) m_{3/2}, \;\; \mathrm{LVS}: \; m_\phi \simeq \left(\frac{m_{3/2}}{M_{\rm pl}}\right)^{1/2} m_{3/2}\,, \tag{177}$$

with ϕ the lightest modulus (e.g., the volume modulus in LVS) and $m_{3/2} = \frac{W_0}{\mathcal{V}} M_{\rm pl}$.

其中最轻的模量为 ϕ (例如 LVS 中的体积模量)，且 $m_{3/2} = \frac{W_0}{\mathcal{V}} M_{\rm pl}$ 。

Because the moduli have gravitational-strength couplings, their decay rates are suppressed by the Planck mass, with

由于模量只以引力强度耦合，它们的衰变率被普朗克质量压低，即

$$\Gamma \simeq \frac{m_\phi^3}{16\pi M_{\rm pl}^2}, \tag{178}$$

and the corresponding reheating temperature from moduli decay is of order

模量衰变对应的再加热温度量级为

$$T_{\rm rh} \simeq \left(\frac{m_\phi}{M_{\rm pl}}\right)^{1/2} m_\phi. \tag{179}$$

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[48] Oversimplified four-dimensional EFT models of axion monodromy often omit the effects of backreaction of monodromy charge, but this is akin to assuming a shift symmetry, as in (173).

[48] 轴子单畴的过度简化四维有效场论模型通常会忽略单畴荷的反作用效应，但这相当于假设了平移对称性，正如文献 (173) 所述。

These properties of the moduli have a number of important post-inflationary implications:

模量的这些性质会带来许多重要的暴涨后宇宙学效应:

- Moduli domination. Because the moduli are light but not relativistic, and because their decay rate is suppressed by powers of the Planck mass, they tend to be very stable, and can dominate the energy density of the universe as in matter domination, with $\rho \sim 1/a(t)^3$. This means that in theories with moduli, inflation is very often followed by a period of moduli domination, unlike the standard Big Bang cosmology.

- 模量主导。由于模量质量很轻但不具有相对论性，且其衰变率被普朗克质量的幂次压低，因此模量通常非常稳定，可以像物质主导那样主宰宇宙的能量密度，满足 $\rho \sim 1/a(t)^3$ 。这意味着，在包含模量的理论中，暴涨结束后通常会进入一段模量主导时期，这和标准大爆炸宇宙学不同。

- Cosmological moduli problem (CMP) [331-333]. In order to avoid spoiling the success of Big Bang nucleosynthesis (BBN), the reheating temperature from moduli decays has to satisfy $T_{\rm rh} \gtrsim 1\text{MeV}$, which implies $m_\phi \gtrsim 30\text{TeV}$. In the naive estimate that $m_\phi \simeq M_{\rm soft} \simeq m_{3/2}$, it would thus be difficult to have $m_\phi \gtrsim 30\text{TeV}$ while imposing $M_{\rm soft} \simeq 1\text{TeV}$ in order to address the hierarchy problem.

- 宇宙学模量问题 (CMP)[331-333]。为了不破坏大爆炸核合成 (BBN) 的成功预言，模量衰变的再加热温度必须满足 $T_{\rm rh} \gtrsim 1\text{MeV}$ ，由此可得 $m_\phi \gtrsim 30\text{TeV}$ 。在 $m_\phi \simeq M_{\rm soft} \simeq m_{3/2}$ 的朴素估计下，为解决等级问题需要满足 $M_{\rm soft} \simeq 1\text{TeV}$ ，此时很难得到 $m_\phi \gtrsim 30\text{TeV}$ 。

In concrete KKLT and LVS scenarios, the soft masses depend on the gravitino mass in different ways, depending on the location of the Standard Model (on D3-branes or D7-branes) and the degree of sequestering of the supersymmetry-breaking sector. For instance as we have seen in the previous section (see Table 2)

在具体的 KKLT 和 LVS 场景中，软质量对引力微子质量的依赖方式并不相同，具体取决于标准模型的位置 (在 D3 膜或 D7 膜上) 以及超对称破缺区的隔离程度。例如我们在前一节已经看到 (参见表 2)

$$\text{KKLT:}\ \ m_\phi \simeq \ln^2\left(\frac{M_{\rm pl}}{m_{3/2}}\right) M_{\rm soft}$$

$$\text{LVS}: \; m_\phi \simeq M_{\text{soft}} \;\; \text{or} \;\; m_\phi \simeq \left(\frac{M_{\text{pl}}}{m_{3/2}}\right)^{1/2} M_{\text{soft}} \tag{180}$$

Therefore, the cosmological moduli problem may be ameliorated in some scenarios. Furthermore, the lack of evidence for supersymmetry at the TeV scale has turned this problem into a feature, namely, moduli domination.

因此，宇宙学模量问题可以在部分场景中得到缓解。此外，TeV 能标尚未发现超对称性的证据反而将这个问题转化为了一个优势，即模量主导本身。

Table 2 Structure of soft supersymmetry-breaking terms (in orders of magnitude) for both KKLT and LVS. The D3 and D7 column labels indicate that the Standard Model is hosted either on D3- branes at a singularity or on D7-branes wrapping a four-cycle with size τ_a

表 2 KKLT 和 LVS 中软超对称破缺项的结构 (量级表示)。D3 和 D7 列标表示标准模型分别位于奇点处的 D3 膜，或缠绕大小为 τ_a 的四周期的 D7 膜上

Soft term	KKLT		LVS	
	D3	D7	D3	D7
$M_{1/2}$	$\frac{m_{3/2}}{\lvert\log\lvert W_0\rvert\rvert}$	$\frac{m_{3/2}}{\lvert\log\lvert W_0\rvert\rvert}$	$\frac{m_{3/2}}{g_s^3\mathcal{V}}$	$\frac{m_{3/2}}{c\tau_a}$
m_α^2	$\frac{m_{3/2}^2}{g_s^3\mathcal{V}}$	$m_{3/2}^2$	$\frac{m_{3/2}^2}{g_s^3\mathcal{V}}$	$\frac{m_{3/2}^2}{(c\tau_a)^2}$
$A_{\alpha\beta\gamma}$	$M_{1/2}$	$M_{1/2}$	$M_{1/2}$	$M_{1/2}$

- Overshoot problem and kination. After inflation, which is thought to occur at relatively high energies, moduli fields will evolve toward the minimum of their scalar potential. Since the vacuum has to describe physics at low energies, there is in general a hierarchy between the initial and final energies. Furthermore, since universal fields including the overall volume and dilaton have runaway minima, it is then possible that the evolution of some modulus field ϕ is energetic enough to pass over the barrier between the physical four-dimensional vacuum and the vacuum at infinity [334]. This difficulty is known as the overshoot problem.

- 过冲问题与动能主宰。一般认为暴涨发生在相对较高的能量，暴涨结束后，模量场会向自身标量势的极小值演化。由于真空需要描述低能物理，初始能量和最终能量之间通常存在等级差。此外，由于整体体积、dilaton 这类全域场具有逃逸极小，部分模量场 ϕ 的演化可能能量足够高，越过物理四维真空和无穷远真空之间的势垒 [334]。这个困难就是所谓的过冲问题。

Fortunately, there are ways to address this problem [335-338]. If the kinetic energy of the evolving modulus dominates over other sources, the resulting period is known as kination. The FLRW equations show that the kinetic energy density during kination dilutes as $\rho \propto 1/a(t)^6$, which is much faster than other sources of energy $(1/a^4$ for radiation or $1/a^3$ for matter). Thus, these other sources will end up dominating, and the corresponding friction term in the scalar field equation

幸运的是，已有方法可以解决该问题 [335-338]。如果演化中模子的动能主导了其他能量源，这一阶段就被称为动能主导时期 (kination)。FLRW 方程表明，动能主导时期动能密度的稀释速度为 $\rho \propto 1/a(t)^6$ ，远快于其他能量源 (辐射为 $(1/a^4$ ，物质为 $1/a^3$)。因此其他能量源最终会占据主导，标量场方程中对应的摩擦项

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{181}$$

is enough to efficiently avoid overshooting. This can be explicitly computed by considering the set of equations as a dynamical system in which for simple enough potentials, like $V \sim e^{-\lambda\phi}$ in the rolling region (such as in LVS), there is an attractor solution that avoids overshooting.

就足以有效避免超调。这可以通过将方程组作为动力学系统显式计算得出: 对于足够简单的势，比如滚动区域的 $V \sim e^{-\lambda\phi}$ (例如 LVS 中的情况)，存在一个可避免超调的吸引子解。

- Dark Radiation. Another consequence of moduli for post-inflationary physics is that the last modulus to decay, rather than the inflaton field itself, is responsible for (the final stage of) reheating. This differs from the simplest inflationary cosmology. The decay products tend to include, in addition to the Standard Model fields, other light bosons resulting from the compactification, such as ultralight axions or dark photons, which may contribute to dark radiation [339, 340][49] . The bounds on dark radiation are very tight. These are parameterized by the effective number of neutrinos $N_{\rm eff}$. It is known experimentally that any contribution to this number from fields beyond the Standard Model is bounded by [342]

- 暗辐射。模子对暴胀后物理的另一个影响是，负责 (末阶段) 再加热的不是暴胀子场本身，而是最后衰变的模子，这和最简单的暴胀宇宙学不同。模子的衰变产物除了标准模型场外，通常还包含紧致化产生的其他轻玻色子，比如超轻轴子或暗光子，它们可以贡献暗辐射 [339, 340][49] 。对暗辐射的限制非常严格，用有效中微子数 $N_{\rm eff}$ 参数化。实验已知，标准模型之外的场对该数值的任何贡献都满足界限 [342]

$$N_{\rm eff}^{\rm exp} = 2.99 \pm 0.17,\ N_{\rm eff}^{\rm SM} = 3.04,\ \Delta N_{\rm eff} < 0.2, \tag{182}$$

leaving almost no room for extra contributions. This is a strong experimental constraint on string theory models, and is also an opportunity. For instance, the axion partner of the volume modulus in LVS may be a candidate for dark radiation, since it is always relativistic, as its mass is of order $m \simeq e^{-c\mathcal{V}^{2/3}}$: see [343] for a recent analysis of dark radiation in LVS.

几乎没有给额外贡献留下空间。这是对弦理论模型的强力实验约束，同时也是一个研究机遇。例如，LVS 中体积模子的轴子伙伴可以成为暗辐射的候选者，因为它始终是相对论性的，质量量级为 $m \simeq e^{-c\mathcal{V}^{2/3}}$: 关于 LVS 中暗辐射的最新分析参见文献 [343]。

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[49] For a recent review with further references, see [7]. Dark radiation constraints from ultralight axions were considered in [341], and found not to become more severe as the number of axions increases.

[49] 如需包含更多参考文献的近期综述，参见 [7]。文献 [341] 研究了超轻轴子带来的暗辐射约束，发现约束强度不会随轴子数量增加而变强。

- Oscillons and moduli stars. As the scalar fields evolve toward their vacuum state, in addition to the coherent oscillations that lead to moduli domination, they may give rise to inhomogeneities known as oscillons or oscillatons (or boson stars): these are non-topological solitonic objects that are unstable but long-lived. Independent of gravity, the structure of the scalar potential away from the minimum may be sensitive to the nonlinear self-interactions of the scalar field. In an expansion around the minimum, the potential can be written as

> - 振荡子与模子星。当标量场向真空态演化时，除了引发模子主导的相干振荡外，还可能产生被称为振荡子、振荡原子 (或玻色子星) 的不均匀性: 它们是非拓扑孤子对象，不稳定但寿命很长。不考虑引力时，偏离极小值的标量势结构会对标量场的非线性自相互作用非常敏感。在极小值附近展开后，势可以写为

$$V = V_0 + m^2\phi^2 + \frac{g\phi^n}{\Lambda^{n-4}} + \dots, \tag{183}$$

with n the first nonvanishing power and Λ a cutoff. If $g < 0$, then the interaction is attractive and tends to give rise to oscillons of mass $M \simeq \Lambda^2/m$ and size $R \sim 1/m$. The mechanism for their formation may be either tachyonic oscillations or parametric resonance. Once gravity is relevant $\left(\Lambda \simeq M_{\mathrm{pl}}\right)$, it contributes to the attraction and gives rise to oscillatons or boson stars (see [344] for recent review and references).

> 其中 n 是第一个非零幂次，Λ 是截断。如果满足 $g < 0$ ，相互作用就是吸引性的，容易形成质量为 $M \simeq \Lambda^2/m$ 、尺度为 $R \sim 1/m$ 的振荡子。其形成机制可以是 tachyonic 振荡，也可以是参数共振。当引力发挥作用 $(\Lambda \simeq M_{\mathrm{pl}})$ 后，引力会增强吸引作用，形成振荡原子或玻色子星 (近期综述及参考文献参见 [344])。

These objects could be formed by the KKLT modulus, or by the small modulus in LVS, but not by the volume or fiber moduli [345,346]. Numerical simulations have been performed illustrating the structure of these inhomogeneities and in particular the spectrum of gravitational waves that they produce. Even though the spectrum varies, the corresponding frequency of the gravitational waves is of order $\omega \simeq 1\mathrm{GHz}$, which is well beyond the reach of LIGO/VIRGO and LISA. This has motivated recent efforts to search for ultra-high-frequency gravitational waves.

> 这些天体可以由 KKLT 模子或 LVS 中的小模子形成，但不能由体积模子或纤维模子形成 [345,346]。已有数值模拟模拟了这些不均匀性的结构，特别是它们产生的引力波谱。尽管谱存在差异，但引力波的对应频率量级为 $\omega \simeq 1\mathrm{GHz}$ ，远超出 LIGO/VIRGO 和 LISA 的探测范围。这推动了近年对超高频率引力波的搜索研究。

High-energy theories, including string theories, furnish other potential sources of gravitational waves that could test energies as high as the GUT scale, $M \simeq 10^{-2}M_{\mathrm{pl}}$. The sensitivity required to probe these frequencies is beyond current technology, but the fact that there are no known astrophysical sources for such high-frequency gravitational waves makes these waves very appealing future targets: a detection would shed light on physics at energies far beyond the reach of colliders and at the same time probe very early cosmology. For a comprehensive review of ultra-high-frequency gravitational waves, see [347].

包括弦论在内的高能理论提供了其他潜在引力波源，可对高达大统一能标的能量进行检验，$M \simeq 10^{-2}M_{\rm pl}$ 。探测这些频率所需的灵敏度超出了当前技术水平，但由于不存在已知的天体物理源能够产生这类高频引力波，这类引力波仍是极具吸引力的未来研究目标: 探测到这类引力波将有助于我们了解对撞机能量范围远不能及的物理，同时也能探究极早期宇宙。关于超高频率引力波的全面综述，参见文献 [347]。

Dark Energy

暗能量

Observations of supernovae and of the cosmic microwave background show that the expansion of the Universe is accelerating. The current dark energy density is

超新星和宇宙微波背景的观测表明，宇宙正在加速膨胀。当前暗能量密度为

$$\rho \approx 10^{-123} M_{\rm pl}^4 \tag{184}$$

with an equation of state relating ρ and pressure p [342]

满足一个联系 ρ 与压强 p 的物态方程 [342]

$$w = \frac{p}{\rho} = -1.03 \pm 0.03. \tag{185}$$

The most straightforward explanation is that the dark energy density comes from a cosmological constant, $p = -\rho$, which is positive but extremely small. This is the origin of the now-standard ΛCDM model for cosmology, where Λ stands for the cosmological constant and CDM for cold dark matter.

最直接的解释是，暗能量密度来自宇宙学常数 $p = -\rho$ ，其数值为正但极小。这就是如今标准宇宙学 ΛCDM 模型的起源，其中 Λ 代表宇宙学常数，CDM 代表冷暗物质。

The cosmological constant problem is the task of explaining why (184) is so small. At present there is no generally accepted dynamical solution to the cosmological constant problem, in quantum field theory or in string theory. However, string theory has provided a framework: anthropic selection in the landscape of flux compactifications [50] .

宇宙学常数问题就是要解释为何 (184) 的数值如此之小。目前，无论在量子场论还是弦论中，宇宙学常数问题都没有得到普遍接受的动力学解。不过弦论已经提供了一个框架: 流量紧化景观 [50] 中的人择选择。

The idea that the cosmological constant might be determined by anthropic selection was advanced by Weinberg, who argued that requiring the formation of galaxies imposes the constraint [350]:

宇宙学常数可能由人择选择确定的观点由温伯格提出，他认为星系形成的要求给出了以下约束 [350]:

$$-10^{-123} \lesssim \rho / M_{\mathrm{pl}}^{4} \lesssim 3 \times 10^{-121}. \tag{186}$$

However, even if one accepts anthropic [51] selection as an approach, it is also necessary to have a family of theories or vacua, among which are some that obey (186) and can be selected.

然而，即使接受人择 [51] 选择作为研究方法，仍然需要存在一族理论或真空，其中有部分满足 (186)、可以被选择出来。

The next key idea was the Bousso-Polchinski [352] observation that com-pactifications admitting many choices of quantized flux furnish a densely spaced ”discretuum” of vacua, potentially including some with ρ near the observed value (cf. [353]). The toy model proposed by Bousso and Polchinski takes the form (in units $M_{\mathrm{pl}}=1$):

下一个核心思想是布索-波尔钦斯基 [352] 的观测结果: 允许多种量子化流量选择的紧化会给出间距密集的真空“离散谱”，其中可能包含 ρ 接近观测值的真空 (参见 [353])。布索和波尔钦斯基提出的 toy model 形式如下 (单位取 $M_{\mathrm{pl}}=1$):

$$\Lambda=\Lambda_{\text {bare }}+\vec{q}^{\top} \cdot \vec{q}, \tag{187}$$

where $\vec{q} \in \mathbb{Z}^{N}$ represents a vector of flux integers. If the bare cosmological constant $\Lambda_{\text {bare }}$ is negative (and, we suppose, of order unity), then finding a configuration with small $|\Lambda|$ from (187) amounts to finding a lattice vector $\vec{q}$ whose length is extremely close to $\sqrt{\left|\Lambda_{\text {bare }}\right|}$. For $N \gg 1$ this is typically possible in principle, but actually finding such a vector is exponentially costly [86].

其中 $\vec{q} \in \mathbb{Z}^{N}$ 代表流量整数构成的向量。如果裸宇宙学常数 $\Lambda_{\text {bare }}$ 为负 (且我们假设量级为 1)，那么从 (187) 中找到一个 $|\Lambda|$ 很小的构型，等价于找到一个长度极接近 $\sqrt{\left|\Lambda_{\text {bare }}\right|}$ 的格点向量 $\vec{q}$ 。对于 $N \gg 1$ ，这在原理上通常是可行的，但实际找到这样一个向量的复杂度是指数级的 [86]。

Bousso and Polchinski did not propose a complete microscopic realization of their scenario in [352], but a framework came soon after, in the form of the landscape of flux vacua in type IIB compactifications, with moduli stabilized as in the KKLT scenario. The fluxes in question are F_3 and H_3 , and the large dimension N of the flux lattice is $N=2 h^{2,1}$, which is of order hundreds in typical Calabi-Yau threefolds.

布索和波尔钦斯基在 [352] 中没有给出该图景完整的微观实现，但不久后就出现了对应框架:IIB 型紧化中的流量真空景观，其模如 KKLT 方案那样被稳定。这里涉及的流量是 F_3 和 H_3 ，流量格点的大维度 N 为 $N=2 h^{2,1}$ ，在典型的卡拉比-丘三维流形中，其量级为数百。

The AdS_4 vacua found in [80] and reviewed in section ”Explicit Constructions” are incarnations of the KKLT scenario, and can have exponentially small cosmological constants. However, the vacua of [80] are not realizations of the Bousso-Polchinski mechanism as written in (187): the bare cosmological constant $\Lambda_{\text {bare }}$ vanishes in the vacua of [80], and N need not be large: examples exist with $N=4$ and 5 in which $|\rho| \ll 10^{-123} M_{\mathrm{pl}}^{4}$. Moreover, the moduli mass scale in [80] is itself extremely small, whereas if one finds small Λ

via (187), it remains possible that all mass scales other than Λ itself are large. Thus, the constructions of [80] constitute progress in achieving scale separation in stabilized vacua, but not in solving the cosmological constant problem.

[80] 中找到、并在“显式构造”章节综述的 AdS_4 真空是 KKLT 方案的具体实现，可以得到指数级小的宇宙学常数。但 [80] 中的真空并不是 (187) 所写的布索-波尔钦斯基机制的实现:[80] 的真空中裸宇宙学常数 Λ_{bare} 为零，且 N 不必很大: 存在 $N = 4$ 等于 5 且满足 $|\rho| \ll 10^{-123} M_{\text{pl}}^4$ 的例子。此外，[80] 中模的质量标度本身极小，而如果通过 (187) 得到小 Λ，除 Λ 本身外所有其他质量标度仍可以保持很大。因此，[80] 的构造在实现稳定真空的标度分离上取得了进展，但并未解决宇宙学常数问题。

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[50] Those who prefer to avoid anthropic arguments are encouraged to propose an alternative in the form of a dynamical explanation of the smallness of the dark energy. For recent attempts see for instance [348, 349].

[50] 不赞同人择论证的研究者需要为暗能量的微小性提出另一种动力学解释。近期相关尝试可参见例如 [348, 349]。

[51] A conceptually similar approach that does not explicitly invoke observers appears in [351].

[51] [351] 中出现了一个概念上类似、且不明确引入观测者的方法。

Although the outlines are in place, a proper implementation of the anthropic approach to the cosmological constant problem will require a better understanding of how to populate the landscape through vacuum decays, and how to define a measure on the string landscape (see, e.g., [354]). Vacuum decays in gravitational theories have been studied for more than 40 years [355-357], and even though these studies were carried out using semiclassical techniques, there is a coherent picture that fits well with string theory. The decay proceeds by means of the nucleation of a bubble of one vacuum in the background of the original vacuum. In the string landscape, there would be a huge number of vacua being constantly nucleated, providing an example of eternal inflation (for a review of eternal inflation, see [358]). In type IIB flux compactifications, the bubble wall would correspond to fivebranes wrapping the relevant three cycles [52] . The Euclidean formalism of [355] then seems to imply that the bubble universe would be an open universe. If true, this would be a concrete prediction of the landscape (see for instance [359]). Populating this landscape could allow one of the bubbles to correspond to a universe with a cosmological constant consistent with current observations.

尽管框架已经搭建完成，要将人择原理方法正确应用于宇宙学常数问题，仍需要更好地理解如何通过真空衰变生成弦景观，以及如何在弦景观上定义测度 (参见例如文献 [354])。引力理论中的真空衰变已经被研究了四十余年 [355-357]，尽管这些研究都采用半经典技术，但已经得到了一幅与弦论契合良好的连贯图像。衰变过程通过在原真空背景中核化新真空泡实现。在弦景观中，会不断核化出数量极多的真空泡，这就是永恒暴涨的一个例子 (永恒暴涨的综述参见 [358])。在 IIB 型通量紧致化中，泡壁对应包裹相关三维闭链的五膜 [52] 。文献 [355] 的欧几里得形式化似乎意味着泡宇宙会是一个开放宇宙。若该结论成立，这会是弦景观的一个具体预言 (参见例如 [359])。生成弦景观的过程可以让其中一个泡对应到宇宙学常数与现有观测一致的宇宙。

Overall, the measure problem and the dynamics of populating the landscape are complex and subtle questions that are not well-understood, but that could have significant impact on our understanding of cosmology. For recent discussions see for instance [360-371].

总体而言，测度问题与生成弦景观的动力学是复杂且精妙的问题，目前尚未被充分理解，但它们会对我们理解宇宙学产生重大影响。近期相关讨论参见例如 [360-371]。

A cosmological constant appears to be the simplest explanation for the dark energy, but the reality may be more complicated. Current observations are also consistent with the idea of quintessence, in which an ultra-light scalar field is slowly rolling today, driving accelerated expansion. Note that even though runaway potentials are generic in string theory, quintessence is not, because the potential needs to be very flat for the field to roll slowly today. There have been several attempts to implement quintessence in flux compactifications: see for instance [372- 375] for early works, and [7, 192, 376-379] for recent discussions of quintessence in string theory.

宇宙学常数似乎是解释暗能量最简单的方案，但实际情况可能更加复杂。现有观测也与精质模型的结论相符: 该模型认为当前存在一个超轻标量场在缓慢滚降，驱动宇宙加速膨胀。需要注意的是，虽然逃逸势在弦论中十分普遍，但精质模型并非如此，因为该模型要求势足够平缓，才能保证标量场到今天仍能缓慢滚降。已经有不少工作尝试在通量紧致化中实现精质模型: 早期工作参见例如 [372-375]，弦论中精质模型的近期讨论参见 [7, 192, 376-379]。

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[52] Ensuring that moduli remain stabilized across a bubble nucleation transition is an obstacle to populating a landscape dynamically in type IIB compactifications.

[52] 在 IIB 型紧致化中，保证模量在泡核化跃迁过程中始终保持稳定，是动态生成弦景观的一个障碍。

Axions

轴子

In an effective field theory, the lightest - and hence, often the most important - fields are those whose mass terms are controlled by symmetries. The four-dimensional effective theories resulting from Calabi-Yau orientifold compactifi-cations of superstring theories always include the massless graviton, protected by general covariance, but also a number of axions [53], i.e., pseudoscalars θ with approximate shift symmetries $\theta \to \theta +$ const. As we have reviewed, moduli masses receive quantum corrections that typically put moduli out of reach of terrestrial and astrophysical measurements: the hard task in moduli stabilization is accurately computing the moduli masses, not establishing that they are generically large compared to Standard Model scales. Finally, gauge bosons and chiral fermions are found in many, but not all, compactifications. In this sense, the generic prediction of Calabi-Yau flux compactifications, in the long-wavelength limit, is four-dimensional Einstein gravity coupled to axion fields.

在有效场论中，质量项受对称性控制的场是最轻的，因此往往也是最重要的。超弦理论经卡拉比-丘定向模紧化后得到的四维有效理论，始终包含广义协变保护下的无质量引力子，同时也包含若干轴子 [53]，即具有近似平移对称性 $\theta \to \theta +$ 常数的赝标量 θ。正如我们已经回顾的，模质量会获得量子修正，这通常会使模超出地球与天体物理测量的可及范围: 模稳定的难点在于精确计算模质量，而非确认模质量一般都远大于标准模型能标。最后，规范玻色子与手征费米子存在于许多 (而非所有) 紧化方案中。在此意义上，卡拉比-丘通量紧化的长波长极限下的一般预言，即四维爱因斯坦引力与轴子场耦合。

Axions are the subject of an immense array of experimental searches: through their interactions with light and matter on Earth, in the sun, in supernovae, in galaxies, and in the CMB, as well as through their gravitational effects on structure formation. Because there are so many paths to improved upper limits (or to a detection), in different environments and relying on different physics and technology, it is clear that axion science will advance rapidly in the coming decades [54]. Combining this state of affairs with the near-inevitability of axions in string theory, we conclude that axions furnish one of the most promising paths to testing string theory.

轴子目前是海量实验搜寻工作的研究对象: 搜寻途径包括轴子与地球、太阳、超新星、星系、宇宙微波背景中的光与物质的相互作用，以及轴子对结构形成的引力效应。由于存在多种不同环境、依托不同物理与技术的方法来改进轴子性质的上限 (或是直接探测到轴子)，未来几十年轴子科学显然会快速发展 [54]。结合这一情况，再加上弦理论中轴子几乎必然存在，我们可以得出结论: 轴子是检验弦理论最具前景的方向之一。

Axions in String Theory

弦论中的轴子

Axions in string theory [55] result from dimensional reduction of p-form potentials on p-cycles. Consider a term in the ten-dimensional action of the form:

弦论中的轴子 [55] 来源于 p-形式势在 p-闭链上的维度约化。考虑十维作用量中如下形式的项:

$$S_{C_p} = \int \mathrm{d}C_p \wedge \star \mathrm{d}C_p. \tag{188}$$

The action (188) is minimized if $\mathrm{d}C_p = 0$, and the equation of motion reads [56]:

当 $\mathrm{d}C_p = 0$ 时作用量 (188) 取最小值，运动方程为 [56]:

$$\Delta C_p = 0, \tag{189}$$

where Δ is the Laplacian acting on p-forms. The solutions to (189) correspond, by Hodge’s theorem, to cohomology classes in $H^p(X,\mathbb{R})$.

其中 Δ 是作用于 p-形式的拉普拉斯算符。根据霍奇定理，(189) 的解对应 $H^p(X,\mathbb{R})$ 上上同调类。

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[53] Some authors reserve the term "axion" for a particular pseudoscalar coupled to QCD, and refer to other shift-symmetric pseudoscalars as "axion-like particles," but we will not follow this usage: we will speak of many axions, one of which is the QCD axion.

[53] 部分学者仅将“轴子”一词指代耦合量子色动力学的特定赝标量，将其他平移对称的赝标量称为“类轴子粒子”，但本文不采用这种区分: 我们将它们统称为轴子，其中之一便是量子色动力学轴子。

[54] See [380] for a review of the physics of axions.

[54] 轴子物理学的综述可参见文献 [380]。

[55] See, e.g., [310, 381 – 383] for early works on axion couplings in string theory.

[55] 弦论中轴子耦合的早期研究可参见例如 [310, 381 – 383] 。

[56] To reach the form (189), one imposes the gauge-fixing condition $\mathrm{d}^\dagger C_p = 0$, where $\mathrm{d}^\dagger := -\star\mathrm{d}\star$ is the adjoint exterior derivative.

[56] 为得到 (189) 的形式，需要施加规范固定条件 $\mathrm{d}^\dagger C_p = 0$ ，其中 $\mathrm{d}^\dagger := -\star\mathrm{d}\star$ 是伴随外微分。

Topologically trivial changes in the field configuration of the potential C_p - i.e., changes $C_p \to C_p + \mathrm{d}A_{p-1}$, with A_{p-1} a $(p-1)$ -form, are gauge redundancies, and in particular the action (188) is invariant. However, a change in the cohomology class of $C_p, C_p \to C_p + c\omega_p$, with $c \in \mathbb{R}$ and with $\omega_p \in H^p(X,\mathbb{Z})$, can be detected by a suitable object charged under C_p . The Wilson line in Maxwell theory is an example for $p = 1$.

势场 C_p 场构型的拓扑平凡改变——即满足 $C_p \to C_p + \mathrm{d}A_{p-1}$ 的改变，其中 A_{p-1} 是 $(p-1)$ -形式——属于规范冗余，作用量 (188) 在此变换下保持不变。然而，当满足 $c \in \mathbb{R}$ 和 $\omega_p \in H^p(X,\mathbb{Z})$ 时，$C_p, C_p \to C_p + c\omega_p$ 的上同调类改变可以被一个携带 C_p 荷的合适探测对象观测到。麦克斯韦理论中的威尔逊线就是 $p = 1$ 的一个例子。

In type IIB string theory, we will focus on the case $p = 4$, with C_4 the Ramond-Ramond four-form. We take ω_a to be a basis of $H^4(X,\mathbb{Z})$, with ω^a the dual homology basis. Euclidean D3-branes wrapping four-cycle D couple to C_4 as in (34), and so writing an integral divisor class $D = d_a\omega^a$ in terms of integers d_a , we have:

在 IIB 型弦论中，我们聚焦于 $p = 4$ 的情形，其中 C_4 是拉蒙德-拉蒙德四形式。我们取 ω_a 作为 $H^4(X,\mathbb{Z})$ 的一组基，ω^a 是对偶同调基。欧几里得 D3 膜缠绕四闭链 D 后按照 (34) 的形式与 C_4 耦合，因此将整除子类 $D = d_a\omega^a$ 用整数 d_a 展开后，我们得到:

$$\frac{1}{2\pi} S_{\mathrm{ED3}} = \mathrm{Vol}(D) + i d_a \int_{\omega^a} C_4. \tag{190}$$

Under $C_p \to C_p + c^a\omega_a$, the Euclidean D3-brane action (190) shifts as

在 $C_p \to C_p + c^a \omega_a$ 变换下，欧几里得 D3 膜作用量 (190) 变为

$$S_{\mathrm{ED3}} \to S_{\mathrm{ED3}} + 2\pi i c^a d_a \tag{191}$$

Integral shifts remain a symmetry even in the presence of Euclidean D3-branes, but the continuous shift symmetry is broken.

即使存在欧几里得 D3 膜，整数平移仍然是一种对称性，但连续平移对称性被破坏。

To examine the corresponding four-dimensional axions, we make the ansatz:

为了研究对应的四维轴子，我们给出如下假设:

$$C_4 = \theta^a(x)\,\omega_a, \tag{192}$$

with ω_a a basis of $H^4(X, \mathbb{Z})$, and find

其中 ω_a 是 $H^4(X, \mathbb{Z})$ 的一组基，最终得到

$$S_{C_p} = -M_{\mathrm{pl}}^2 \int \mathrm{d}^4x\sqrt{-g}\partial_\mu\theta^a\partial_\nu\theta^b \int_X \omega_a \wedge \star\omega_b. \tag{193}$$

The pairing

该配对

$$\int_X \omega_a \wedge \star\omega_b = \frac{1}{2}K_{ab}^{\mathrm{tree}} = \frac{1}{4}\frac{\partial}{\partial\tau_a}\frac{\partial}{\partial\tau_b}K_{\mathrm{tree}} = -\frac{1}{2}\frac{\partial}{\partial\tau_a}\frac{\partial}{\partial\tau_b}\log(\mathcal{V}) \tag{194}$$

thus defines the matrix of kinetic couplings of the axions θ^a . In (194) we have compared to the tree-level kinetic terms derived in section ”Tree Level”, using (46) and (49), and the derivatives are to be evaluated using (51).

由此定义了轴子的动能耦合矩阵 θ^a 。(194) 与“树图阶”章节中利用 (46) 和 (49) 推导得到的树图动能项一致，导数需要通过 (51) 计算。

The axions θ^a have no nonderivative interactions to any order in the g_s and α' expansions: their potential arises exclusively from non-perturbative effects. This structure is a consequence of the ten-dimensional gauge symmetry, and does not rely on supersymmetry. Moduli, in contrast, do generally receive contributions to their potential at (almost) all orders in both perturbative expansions.

轴子 θ^a 在 g_s 展开和 α' 展开的任意阶都没有非导数相互作用: 它的势完全来自非微扰效应。该结构是十维规范对称性的结果，不依赖超对称。与之相反，模的势通常会在两种微扰展开的 (几乎) 所有阶都获得贡献。

At first glance one might guess that perturbative quantum effects should be easier to compute than non-perturbative quantum effects - if so, the moduli potential would be more accessible to theoretical understanding than the axion potential. It is true that some of the leading perturbative corrections to the effective action are well-understood, and it is also true that a genuinely exact and systematic computation of the non-perturbative potential for axions has not yet been performed. Even so, the axion shift symmetries are so powerful that at the time of writing, much more is known about the structure of the axion potential, and the resulting physics at low energies, than about the corresponding questions for moduli.

乍看之下，我们会认为微扰量子效应比非微扰量子效应更容易计算——果真如此的话，模势会比轴子势更易于从理论上理解。诚然，有效作用量的某些领头阶微扰修正已经得到了很好的理解，而且目前确实尚未对轴子的非微扰势完成真正精确且系统的计算。尽管如此，轴子平移对称性的约束力极强，截至撰文之时，我们对轴子势的结构及其带来的低能物理的了解，远多于对模对应问题的了解。

Kreuzer-Skarke Axiverse

克罗伊策-斯卡克轴子宇宙

Let us try to understand this state of affairs in the well-studied case of C_4 axions, which result from the expansion $C_4 = \theta^a \omega_a$ with ω_a a basis for the orientifold-even subspace $H^4_+(X_6, \mathbb{Z})$. The actions S of the Euclidean D3-branes that give the dominant potential terms are determined by the volumes of the wrapped four-cycle D , up to logarithmic corrections from Pfaffians involving the other moduli:

让我们在已得到充分研究的 C_4 轴子情形中理解这一情况，这类轴子由展开式 $C_4 = \theta^a \omega_a$ 得到，其中 ω_a 是定向偶对称子空间 $H^4_+(X_6, \mathbb{Z})$ 的一组基。给出主导势项的欧几里得 D3 膜的作用量 S 由被缠绕四维闭链 D 的体积决定，相差来自包含其他模的普法夫项的对数修正:

$$\mathrm{Re}(S) = 2\pi\,\mathrm{Vol}(D) - \log(\mathcal{A}(z_i, \tau)). \tag{195}$$

If one makes the further assumption that supersymmetry is weakly broken by the expectation value W_0 of the flux superpotential, then the dominant Euclidean D3- branes are those that contribute to the superpotential [57] . These Euclidean D3-branes wrap holomorphic four cycles, which are calibrated by the Kähler form J and so have volumes that are given algebraically in terms of the data of the Kähler parameters and intersection numbers:

如果进一步假设超对称被通量超势的期望值 W_0 弱破缺，那么对超势 [57] 有贡献的就是主导欧几里得 D3 膜。这些欧几里得 D3 膜缠绕全纯四维闭链，闭链由凯勒形式 J 校准，因此其体积可通过凯勒参数数据和相交数以代数形式给出:

$$\mathrm{Vol}(D_a) = \frac{1}{2}\int_{D_a} J \wedge J = \frac{1}{2}\kappa_{abc} t^b t^c. \tag{196}$$

The geometric data of four-cycle volumes is then readily accessible. In contrast, computing the volume of a non-holomorphic four-cycle would require knowing, and integrating, the volume form determined by the Calabi-Yau metric: this could be attempted by building on the metrics obtained in [385-392], but has not been carried out.

四维闭链体积的几何数据因此可以很容易得到。与此相对，计算非全纯四维闭链的体积需要先知道卡拉比-丘度量确定的体积形式并对其积分: 这可以基于文献 [385-392] 中得到的度量尝试，但尚未完成。

In view of the results of section "Non-perturbative Superpotential", computing W_{ED3} at a given point in $T^\star$ in Kähler moduli space involves enumerating the smallest effective divisors at $T^\star$ that support two fermion zero modes, as in (37). Thus, the leading exponentials are determined by knowing topological and geometric data. From the leading exponentials alone, one cannot compute the exact axion potential, but nonetheless one can obtain enough information to make meaningful comparisons to experiments.

根据“非微扰超势”一节的结果，在凯勒模空间中 $T^\star$ 的给定点计算 W_{ED3}，需要枚举 $T^\star$ 上支持两个费米零模的最小有效除子，如 (37) 所示。因此，主导指数由拓扑与几何数据决定。仅靠主导指数无法得到精确轴子势，但仍足以获得有意义的实验对比信息。

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[57] We set aside the exotic possibility of parametrically large recombination of non-holomorphic cycles [384].

[57] 我们不考虑非全纯闭链参数化大重组的特殊可能性 [384]。

In summary, the four-dimensional effective action for axions can be computed with knowledge of the topological data of a Calabi-Yau orientifold, together with the specification of a point $T^\star$ in Kähler moduli space. In a complete top-down derivation, one would compute the potential for Kähler moduli, find its minimum $T_{\min}$, and study axion couplings there. At present Kähler moduli stabilization can be performed only in certain lamppost regions of moduli space, where, for example, the KKLT or LVS approaches are applicable. One approach is to fix a scenario and study the axion couplings, as done in [393] for LVS [58] .

总而言之，轴子的四维有效作用量可以结合卡拉比-丘定向偶的拓扑数据，以及凯勒模空间中一个点 $T^\star$ 的设定计算得到。在完整的自上而下推导中，我们需要计算凯勒模的势，找到其极小值 $T_{\min}$，并研究该处的轴子耦合。目前仅能在模空间的某些“路灯区域”完成凯勒模稳定，例如 KKLT 或 LVS 方法适用的区域。一种研究方式是固定一个场景并研究轴子耦合，正如文献 [393] 对 LVS 所做的 [58] 。

Alternatively, one can take advantage of the fact that computing the data of axion effective theories is feasible in a much broader swath of moduli space than the lampposts where moduli stabilization is understood. One can thus aim to characterize axion couplings in string theory by searching for results that hold everywhere in Kähler moduli space or that are at least approximately independent of location [59] .

另一种方式我们可以利用这一事实: 相较于模稳定已被理解的路灯区域，轴子有效理论数据的计算在模空间大得多的范围内都是可行的。因此我们可以通过搜索在整个凯勒模空间都成立、或至少近似不依赖位置 [59] 的结果，来刻画弦论中的轴子耦合。

More precisely, one seeks results that hold, or hold to some level of approximation, everywhere in the subregion of moduli space where computations are controlled. Near the walls of the extended Kähler cone, α' corrections to the effective action are large, and the axion kinetic term is not necessarily well-approximated by the couplings $\int \omega \wedge \star \omega$ that result from the leading-order term $\int \mathrm{d}C_4 \wedge \star \mathrm{d}C_4$ in the ten-dimensional action. Moreover, near loci where divisors shrink, the associated instanton series are poorly controlled, and the axion potential is not a sum of exponentially small periodic terms.

更准确地说，我们寻找在可控制计算的模空间子区域内处处严格成立、或在一定近似下成立的结果。在扩展凯勒锥的边界附近，α' 对有效作用量的修正很大，轴子动能项不一定能很好地由十维作用量领头项 $\int \mathrm{d}C_4 \wedge \star \mathrm{d}C_4$ 导出的耦合 $\int \omega \wedge \star \omega$ 近似。此外，在除子收缩的轨迹附近，相关瞬子级数难以控制，轴子势也不再是指数小周期项的和。

Following [395], we define the stretched Kähler cone as the subregion of the Kähler cone in which all effective curves have volume ≥ 1 :

遵循文献 [395]，我们将拉伸凯勒锥定义为凯勒锥中所有有效曲线体积满足 ≥ 1 的子区域:

$$\mathcal{K} := \left\{ J \in H^{1,1}(X_6, \mathbb{R}) \mid \int_{\mathcal{C}} J > 1 \forall \mathcal{C} \right\}, \tag{197}$$

where the condition is imposed for all effective curves $C \in H_2(X_6, \mathbb{Z})$, and we work in units of $\ell_s^2 = (2\pi)^2 \alpha'$.

其中该条件适用于所有有效曲线 $C \in H_2(X_6, \mathbb{Z})$ ，我们以 $\ell_s^2 = (2\pi)^2 \alpha'$ 为单位计算。

Within the stretched Kähler cone, and in the approximation that the scale of supersymmetry breaking is small compared to the cutoff of the effective theory, the axion kinetic term is determined by classical geometric data, and the dominant potential terms breaking the PQ symmetries arise from the Euclidean D3-brane superpotential W_{ED3} . Thus, a systematic computation of the leading terms in the effective theory of C_4 axions [60] reduces to an algorithmic procedure in computational geometry. In the case that the Calabi-Yau threefold X is a hypersurface in a toric variety, this procedure is actually feasible, and can even be automated: see the discussion in section "Computational Advances".

在拉伸凯勒锥内部，且在超对称破缺尺度远小于有效理论截断的近似下，轴子动能项由经典几何数据决定，破缺 PQ 对称性的主导势项来自欧几里得 D3 膜超势 W_{ED3} 。因此，对 C_4 轴子 [60] 有效理论中领头项的系统计算可归约为计算几何中的算法过程。当卡拉比-丘三维流形 X 是有托 varieties 中的超曲面时，该过程实际可行，甚至可以自动化: 参见“计算进展”一节的讨论。

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[58] A recent treatment appears in [394].

[58] 近期的相关研究可参见 [394]。

[59] Alternatively, one can pursue constraints that result from sampling the moduli space according to a natural measure.

[59] 或者，我们可以根据自然测度对模空间采样，进而得到约束条件。

The Kreuzer-Skarke axiverse [395] is the resulting ensemble of axion effective theories from Calabi-Yau threefold hypersurface compactifications of type IIB string theory, with $1 \leq h^{1,1} \leq 491$. A striking property of this ensemble is that the cycle volumes manifest hierarchies that are polynomial in $h^{1,1}$. An intuitive explanation is that complicated topology does not easily fit into a small (and Ricci-flat) space, while a quantitative explanation in terms of systems of linear inequalities is given in [395]. One empirical result is that - at a reference location in moduli space, the tip of the stretched Kähler cone - certain divisor volumes [61] scale as

Kreuzer-Skarke 轴宇宙 [395] 是 IIB 型弦论经卡拉比-丘三维流形超曲面紧化后得到的轴子有效理论系综，满足 $1 \leq h^{1,1} \leq 491$ 。该系综一个引人注目的性质是，闭链体积表现出关于 $h^{1,1}$ 的多项式层级结构。直观的解释是，复杂拓扑很难嵌入小的 (且里奇平坦的) 空间，[395] 已经给出了基于线性不等式系统的定量解释。一个实验结果是: 在模空间的参考点——拉伸凯勒锥顶点，某些除子体积 [61] 满足标度关系

$$\operatorname{Vol}(D_A) \propto \left(h^{1,1}\right)^4 , \tag{198}$$

though it should be understood that the scatter around (198) is significant [395].

但需要注意，(198) 周围的散射十分显著 [395]。

As a result of (198), the Kreuzer-Skarke axiverse has important qualitative differences from that envisioned in the first discussions of the string axiverse [396]. In both cases there are hundreds of axions ϕ_a with masses m_a spanning a huge range of scales [62] , but the distributions of decay constants f_a are very different. The original scenario of [396] proposed decay constants clustered around a high scale:

受 (198) 的影响, Kreuzer-Skarke 轴宇宙与弦轴宇宙最初讨论中设想的场景存在重要定性差异 [396]。两种场景中都存在数百个轴子 ϕ_a ，其质量 m_a 跨越巨大的尺度范围 [62] ，但衰变常数 f_a 的分布却截然不同。[396] 的原始场景提出衰变常数聚集在高尺度附近:

$$\text{String Axiverse [396]: } f_a \sim M_{\mathrm{GUT}} \approx 2 \times 10^{16} \mathrm{GeV} . \tag{199}$$

In simple Calabi-Yau compactifications that have only a single axion, f_a is indeed typically large, and so an axiverse modeled as $N \gg 1$ copies of such single axions would have a narrow distribution of decay constants f_a . However, it turns out that topologically complex Calabi-Yau threefolds manifest geometric hierarchies, as in (198). As a consequence, the decay constants f_a in the Kreuzer-Skarke axiverse spread over a wide range of scales, with

在仅含单个轴子的简单卡拉比-丘紧化中，f_a 通常确实很大，因此将轴宇宙建模为这类单轴子的 $N \gg 1$ 拷贝会得到窄的衰变常数 f_a 分布。但事实证明，拓扑复杂的卡拉比-丘三维流形会表现出 (198) 中的几何层级结构。因此，Kreuzer-Skarke 轴宇宙中的衰变常数 f_a 分布在很宽的尺度范围内，满足

$$\text{Kreuzer-Skarke Axiverse [395]: } \mathcal{O}\left(10^{9}\right) \text{ GeV } \lesssim f_a \lesssim \mathcal{O}\left(10^{16}\right) \text{ GeV.} \tag{200}$$

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[60] The masses and couplings of the saxions $\tau_a = \mathrm{Vol}(D_a)$ vary across the Kähler moduli space. A reasonable expectation is that near the walls of the Kähler cone, competition among perturbative corrections to the Kähler potential, as in (97), could lead to stabilization of the saxions, though computing the saxion potential in detail is presently out of reach. If the saxions are stabilized by perturbative effects, they will be much heavier than the axions, and will be comparatively decoupled from low-energy phenomena.

[60] 萨克斯子的质量和耦合 $\tau_a = \mathrm{Vol}(D_a)$ 随凯勒模空间变化。一个合理的推测是，在凯勒锥壁附近，如 (97) 所示，凯勒势的微扰修正之间的竞争可能会稳定萨克斯子，不过目前还无法详细计算萨克斯子势。如果萨克斯子由微扰效应稳定，其质量会远大于轴子，相对来说会解耦低能现象。

[61] More precisely, in every basis of $H_4(X, \mathbb{Z}$, at least one (and often, more than one) basis element has size at least as large as (198): see [395] for further details.

[61] 更准确地说，在 $H_4(X, \mathbb{Z}$ 的任意一组基中，至少有一个 (通常不止一个) 基元的大小不小于 (198): 更多细节参见 [395]。

[62] Light axions are both theoretically natural, because their mass originates from a non-perturbative effect, and experimentally allowed: light parity-even spinless fields (i.e., scalars) are strongly constrained by experimental limits on fifth forces, but light pseudoscalars are not.

[62] 轻轴子在理论上是自然的，因为其质量源自非微扰效应，同时也符合实验允许范围: 轻的偶宇称零自旋场 (即标量场) 受到第五力实验极限的严格约束，但轻赝标量场不受此约束。

This finding changes the axion phenomenology as well as the prospects for detection, as we now explain.

这一结论改变了轴子唯象学，也改变了探测前景，我们接下来对此进行说明。

Strong CP Problem

强 CP 问题

One important issue that can be understood in this approach is the strong CP problem. The neutron electric dipole moment is so small that it has never been measured. In quantum field theory without gravity, its smallness can be explained, as suggested by Peccei and Quinn (PQ), by introducing an axion that dynamically relaxes the CP-breaking θ -angle of QCD. However, this solution is very sensitive to quantum gravity corrections: even rather high-dimension Planck-suppressed operators spoil the PQ mechanism. This weakness of the PQ mechanism is called the quality problem.

该方法可以阐释的一个重要问题就是强 CP 问题。中子电偶极矩极小，至今尚未被测量到。在无引力的量子场论中，Peccei 和 Quinn(PQ) 提出，其微小性可以通过引入轴子来解释: 轴子会动态弛豫 QCD 中破坏 CP 的 θ 角。但该解对量子引力修正非常敏感: 即便是高维普朗克压低算符也会破坏 PQ 机制。PQ 机制的这一缺陷被称为质量问题。

Because the success or failure of the PQ mechanism for solving the strong CP problem hinges on quantum gravity effects, one should compute these effects directly in an ultraviolet completion of gravity. This was achieved in [397], in the setting of type IIB compactifications on orientifolds of Calabi-Yau hypersurfaces, in the geometric regime. If QCD is realized on a stack of D7-branes wrapping a four-cycle D_{QCD} , the QCD axion is

由于 PQ 机制解决强 CP 问题的成败依赖于量子引力效应，因此应当在引力的紫外完备理论中直接计算这些效应。这一工作在文献 [397] 中完成，研究设定是几何区域内 Calabi-Yau 超曲面的定向模化上的 IIB 型紧致化。若 QCD 实现于缠绕四维闭链 D_{QCD} 的一堆 D7 膜上，则 QCD 轴子为

$$\theta_{\mathrm{QCD}} = \int_{D_{\mathrm{QCD}}} C_4. \tag{201}$$

The terms in the effective action that spoil PQ quality arise from instantons - specifically, Euclidean D3-branes - that carry PQ charge. Applying the methods reviewed in section ”Computational Advances”, one can enumerate the leading Euclidean D3-brane superpotential terms, which for a four-cycle D_A take the form:

有效作用量中破坏 PQ 质量的项来自瞬子——具体来说是欧几里得 D3 膜——它们携带 PQ 荷。应用“计算进展”一节回顾的方法，可以枚举主导的欧几里得 D3 膜超势项，对于四维闭链 D_A ，其形式为:

$$W_{\mathrm{ED3}} = \mathcal{A}_A e^{-2\pi Q_A^a T_a}. \tag{202}$$

The importance of (202) at a given point in Kähler moduli space falls exponentially with $\mathrm{Vol}\left(D_A\right) = \mathrm{Re}\left(T_A\right)$. Inside the stretched Kähler cone - i.e., in the region of moduli space where the α' expansion is under control - the four-cycle volumes $\mathrm{Vol}\left(D_A\right)$ are hierarchical: a few are $\approx \mathcal{O}\left(1\right) \ell_s^4$ but a majority scale as in (198). The resulting mass of the QCD axion, and hence the shift of the QCD θ -angle $\overline{\theta}$ away from the CP-conserving value $\overline{\theta} = 0$, was computed [63] in a large ensemble of explicit geometries in [397], with the result:

在凯勒模空间的给定点，(202) 的重要性随 $\mathrm{Vol}\left(D_A\right) = \mathrm{Re}\left(T_A\right)$ 呈指数下降。在拉伸的凯勒锥内部——即模空间中 α' 展开可控的区域——四维闭链体积 $\mathrm{Vol}\left(D_A\right)$ 满足分层结构: 少数体积为 $\approx \mathcal{O}\left(1\right) \ell_s^4$ ，而大多数体积符合 (198) 的标度。文献 [397] 在大量显式几何样本中 [63] 计算得到了最终 QCD 轴子的质量，以及 QCD θ 角 $\overline{\theta}$ 偏离 CP 守恒值 $\overline{\theta} = 0$ 的偏移，结果为:

$$\overline{\theta} \propto \exp\left(-cN^4\right), \tag{203}$$

with $N = h^{1,1}$ the number of axions, and equivalently the number of Kähler moduli. We reiterate that (203) contains explicit upper bounds on the leading quantum gravity contributions to $\overline{\theta}$ [64]. Thus, in models with $N \gtrsim 15$ axions, everywhere in moduli space where the α' expansion is under control, the strong CP problem is solved by the Peccei-Quinn mechanism.

其中 $N = h^{1,1}$ 是轴子的数量，等价于凯勒模的数量。我们要重申，(203) 给出了 $\bar{\theta}$[64] 主导量子引力贡献的明确上界。因此，在包含 $N \gtrsim 15$ 个轴子的模型中，模空间内所有 α' 展开可控的区域，佩西-奎因机制都可以解决强 CP 问题。

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[63] The phenomenological scaling of (198) helps explain the result of [397], but was not used as an input in [397]: the four-cycle volumes were directly computed in each case.

[63] (198) 的唯象标度有助于解释 [397] 的结果，但它并未被用作 [397] 的输入: 四维闭链体积是在每个情形中直接计算得到的。

It does not follow that (type IIB) string theory predicts that the strong CP problem is automatically solved in general, because we have no knowledge of $\bar{\theta}$ in the regime where cycles are small and instanton corrections are large. Rather, one can say that Calabi-Yau orientifold compactifications of type IIB string theory, in the geometric regime, furnish an ultraviolet completion of gravity that exhibits an automatic solution of the strong CP problem. Although this solution applies only if $h^{1,1} \gtrsim 15$, the overwhelming majority of known Calabi-Yau threefolds do have $h^{1,1} > 15$[65] .

这并不意味着 (IIB 型) 弦论预言强 CP 问题一般情况下会自动解决，因为我们对闭链很小、瞬子修正很大的区域中的 $\bar{\theta}$ 尚无认知。更准确的说法是，几何区域内 IIB 型弦论的 Calabi-Yau 定向模紧致化提供了引力的紫外完备，该理论中强 CP 问题会自动得到解决。尽管该解仅在 $h^{1,1} \gtrsim 15$ 的条件下成立，但绝大多数已知的 Calabi-Yau 三维流形都满足 $h^{1,1} > 15$[65] 。

Axion-Photon Couplings

轴子-光子耦合

An impressive collection of experiments are searching for axions through their couplings to the photon, for example, through X-ray spectrum oscillations (see, e.g., [398-400]), CMB birefringence, or production in the Sun or the Milky Way. For N canonically normalized axions φ_a with decay constants f_a , the couplings can be written:

已有大量实验通过轴子与光子的耦合搜寻轴子，例如通过 X 射线谱振荡 (参见例如文献 [398-400])、宇宙微波背景双折射，或是观测太阳与银河系内的轴子产生过程。对于 N 正则归一化轴子 φ_a (其衰变常数为 f_a)，耦合可写为:

$$\mathcal{L} \supset -\frac{1}{4} g_{a\gamma\gamma} \varphi^a F_{\mu\nu} \widetilde{F}^{\mu\nu}, \tag{204}$$

with

其中

$$g_{a\gamma\gamma} = \frac{\alpha_{\mathrm{EM}}}{2\pi f_a}\Theta_a \text{ (no sum).} \tag{205}$$

where Θ_a is a dimensionless coupling that results from a mixing angle.

Θ_a 是由混合角产生的无量纲耦合。

Let us temporarily suppose that the mixing angles Θ_a are $\mathcal{O}(1)$, or at least are not hierarchically small. Then the finding that in Calabi-Yau compactifications with many axions [395, 401], the decay constants spread down to small scales, $f \sim \mathcal{O}\left(10^9\right)$ GeV , as in (200), suggests that such theories could easily be ruled out by near-future limits on axion-photon couplings.

我们暂时假设混合角 Θ_a 为 $\mathcal{O}(1)$ ，或至少不是等级式小量。那么，在含多个轴子的卡拉比-丘紧致化中 [395, 401]，衰变常数会延伸至小尺度 $f \sim \mathcal{O}\left(10^9\right)$ GeV ，如式 (200) 所示，这一结果表明这类理论很容易被不久将来的轴子-光子耦合观测极限排除。

It turns out, however, that the mixing angles Θ_a are not of order unity [341, 402]. Two mechanisms are responsible: first, the kinetic matrix is very sparse, as a result of the structure of divisor intersections [402]. Second, Euclidean D3-branes wrapping the four-cycle that supports QED generate a small mass scale, and any axions lighter than this light threshold have hierarchically suppressed couplings to the photon [341].

但事实证明，混合角 Θ_a 并不为一阶量级 [341, 402]。这由两种机制导致: 首先，受除子相交结构的影响，动能矩阵十分稀疏 [402]；其次，包裹支撑量子电动力学四维闭链的欧几里得 D3 膜会产生一个小质量标度，所有轻于这个轻阈值的轴子，其与光子的耦合都会受到等级式压低 [341]。

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[64] The technical assumptions entering (203) are laid out in [397], and are fairly modest: supersymmetry breaking should be well below the compactification scale, and there should be no light states charged under QCD except those from the Standard Model and their superpartners. [65] Strictly speaking, this statement applies to counts of Calabi-Yau threefolds that are potentially inequivalent, but have not yet been proven to be distinct: cf. [136].

[64] 式 (203) 涉及的技术假设已在文献 [397] 中说明，这些假设相当温和: 超对称破缺的能标远低于紧致化能标，且除标准模型及其超对称伙伴外，不存在 QCD 下带电的轻态。[65] 严格来说，该表述适用于计数潜在不等价、但尚未被证明互异的卡拉比-丘三维流形: 参见文献 [136]。

The picture that emerges is of an axiverse that is mostly dark, but in which a handful of axions have couplings to photons that put them in reach of ongoing observations [380,403-405].

最终得到的轴子图景是: 轴子宇宙中的大部分轴子都是暗的，但仍有少量轴子与光子的耦合足够强，可被当前观测探测到 [380, 403-405]。

Black Hole Superradiance

黑洞超辐射

Black hole superradiance is another phenomenon that connects the string axiverse to observations [406,407]. An axion field whose Compton wavelength is comparable to the Schwarzschild radius of a Kerr black hole will develop a superradiant instability and extract angular momentum from the black hole, unless nonlinear self-interactions of the axion disrupt the development of a condensate. Thus, upper bounds on the spins of astrophysical black holes set limits on the existence of axions in a range of mass m and interaction strength (i.e., decay constant) f . These limits do not involve any cosmological assumptions: the axions need not be produced in cosmic history, or persist as a relic population. The axions are created from the vacuum by the black hole, so what is tested is whether such fields are present in the Lagrangian. For stellar-mass black holes, for which the data is better at present, the exclusions are in the vicinity of $m \approx 10^{-10}\text{eV}$, and apply for $f \gtrsim 10^{14}\text{GeV}$.

黑洞超辐射是将弦论轴子宇宙学与观测联系起来的另一种现象 [406,407]。若轴子场的康普顿波长与克尔黑洞的史瓦西半径相当，就会产生超辐射不稳定性，从黑洞提取角动量——除非轴子的非线性自相互作用会破坏凝聚态的形成。因此，天体物理黑洞自转的上限对质量范围 m 、相互作用强度 (即衰变常数) f 区间内轴子的存在给出了限制。这些限制不涉及任何宇宙学假设: 轴子不必在宇宙演化中产生，也不必作为遗迹粒子留存。轴子是黑洞从真空中产生的，因此该过程检验的是这类场是否存在于拉格朗日量中。对于目前观测数据更完善的恒星质量黑洞，排除范围落在 $m \approx 10^{-10}\text{eV}$ 附近，适用于 $f \gtrsim 10^{14}\text{GeV}$ 。

Black hole superradiance in string theory was studied in [401]: the spectrum of C_4 masses and decay constants was computed in an ensemble of type IIB compactifications on Calabi-Yau threefold hypersurfaces, and were compared to measured black hole spins. For this purpose the Kähler moduli were taken at certain reference locations, such as the tip of the stretched Kähler cone. A considerable fraction of geometries at these points in moduli space are excluded by observations.

弦论中的黑洞超辐射已在文献 [401] 中得到研究: 该研究计算了 IIB 型弦论在卡拉比-丘三维超曲面上紧化的系综中，C_4 的质量与衰变常数谱，并将其与观测得到的黑洞自转做对比。为了完成该研究，凯勒模被固定在特定参考位置，例如拉伸凯勒锥的顶点。模空间中这些位置上相当一部分几何都被观测排除。

Computational Advances

计算进展

The study of flux compactifications rests on computation. To construct a Calabi-Yau threefold X_6 and determine the leading-order data of the $\mathcal{N} = 2$ effective theory that results from a type IIB compactification on X_6 , one can start by identifying a suitable ambient space, such as a projective space or a more general toric variety; defining the Calabi-Yau as a subvariety; and computing the Hodge numbers and intersection numbers. One then determines the region of moduli space where this description is valid, for example, by computing the Kähler cone associated with the given geometric phase. The exact prepotential for the vector multiplet moduli space is obtained by computing the periods of the $(3,0)$ form on X_6 , while computing the periods of the $(3,0)$ form on the mirror $\widetilde{X}_6$ and making use of mirror symmetry furnishes a subset of non-perturbative corrections to the hypermultiplet moduli space. Proceeding to an $\mathcal{N}=1$ compactification requires a further series of computations. One identifies an orientifold involution, makes a choice of quantized fluxes, evaluates the flux superpotential in terms of the periods, and enumerates terms in the non-perturbative superpotential by computing the topology of effective divisors.

通量紧致化的研究依赖于计算。要构造一个卡拉比-丘三维流形 X_6 ，并确定在 X_6 上进行 IIB 型紧致化后得到的 $\mathcal{N}=2$ 有效理论的领头阶数据，我们可以先从确定合适的环境空间开始，例如射影空间或更一般的托里克簇；将卡拉比-丘定义为子簇；再计算霍奇数和相交数。随后我们确定该描述有效的模空间区域，例如通过计算给定几何相位关联的凯勒锥。矢量多重态模空间的精确预势可以通过计算 X_6 上 $(3,0)$ 形式的周期得到，而计算镜像 $\widetilde{X}_6$ 上 $(3,0)$ 形式的周期并利用镜像对称性，可以给出超多重态模空间的一部分非微扰修正。进行 $\mathcal{N}=1$ 紧致化还需要一系列额外计算: 我们要确定定向对合反演，选定量子化通量，用周期表示通量超势，并通过计算有效除子的拓扑来列举非微扰超势中的项。

The above computation yields that part of the data of the effective theory that is either classical, holomorphic, or both - in particular, the Käher potential at leading order in g_s and α' and the classical plus non-perturbative superpotential. Approximating the effective theory using only classical plus holomorphic data is a useful starting point at weak coupling and large volume, where perturbative corrections are small. But it is also important to recognize the disparity between the task of computing classical plus holomorphic data, and that of computing perturbative corrections. The steps in the former undertaking, as enumerated above, are all well-specified computations in algebraic geometry and topology. In some cases these might be intractable, or become expensive when the dimension of moduli space is large, but they can at least be stated precisely to a person (or computer) unfamiliar with string theory. Moreover, the questions are generally algebraic or topological in character, rather than analytic.

上述计算得到有效理论中经典或全纯 (或二者兼具) 的那部分数据——具体来说，就是 g_s 和 α' 下领头阶的凯勒势，以及经典加非微扰的超势。在微扰修正很小的弱耦合和大体积场景下，仅用经典加全纯数据近似有效理论是一个实用的出发点。但我们也需要认识到，计算经典加全纯数据的任务，和计算微扰修正的任务并不相同。前文列举的前者的步骤，都是代数几何和拓扑中明确规定好的计算。在某些情况下这些计算可能难以处理，或是在模空间维数较大时计算成本很高，但它们至少可以向不熟悉弦论的人 (或计算机) 精确表述。此外，这类问题本质上通常是代数或拓扑问题，而非分析问题。

In contrast, string perturbation theory is incompletely formulated, especially in general flux backgrounds. Sigma model perturbation theory is hardly better, and evaluating curvature corrections eventually requires computing the curvature of the Ricci-flat metric. Significant progress in characterizing perturbative corrections may require fundamental advances in string theory, and at a minimum will require the application of expertise in highly technical areas of string theory. In summary, the task of computing perturbative corrections remains a physics problem, whereas that of computing the classical plus holomorphic data has been sharpened - by efforts beginning in the early days of supergravity and string theory - into a mathematics problem.

相比之下，弦微扰论的表述并不完整，在一般通量背景下尤其如此。sigma 模型微扰论也好不到哪去，计算曲率修正最终还需要计算里奇平坦度量的曲率。要在刻画微扰修正上取得显著进展，可能需要弦论的基础性突破，至少也需要用到弦论极高技术领域的专业知识。总而言之，计算微扰修正仍然是一个物理问题，而计算经典加全纯数据的任务，从超引力和弦论发展早期就开始不断打磨，如今已经成为一个数学问题。

The past decade has seen extraordinary progress in solving this mathematics problem, and doing so automatically, at scale, and in the general case. Approaches to the problem can be usefully classified into three generations. In the first, pencil and paper were the dominant tool, so the number of relevant moduli was necessarily very small: examples include classic works on mirror symmetry for two-parameter models [408, 409]. The second generation is characterized by increasingly sophisticated computer assistance, but with algorithms whose expense grows exponentially in the number of moduli, so that $\mathcal{O}(10)$ moduli is the upper limit (e.g., [410]). At this stage some codes were purpose-built for research in string theory (e.g., cohomCalg [411]), while others used general-purpose computational geometry and computational algebra software such as Sage and PALP [412].

过去十年中，在求解这个数学问题、并实现通用化大规模自动化求解方面，已经取得了非凡的进展。这个问题的研究方法可以清晰地划分为三代。第一代中，纸笔是主要工具，因此相关模的数量必然非常少: 两参数模型镜像对称的经典研究就是例子 [408, 409]。第二代的特点是计算机辅助日益成熟，但算法的计算成本随模的数量指数增长，因此 $\mathcal{O}(10)$ 个模就是上限 (例如 [410])。在这一阶段，部分代码是专为弦论研究打造的 (例如 cohomCalg[411]，另一部分则使用 Sage 和 PALP 这类通用计算几何与计算代数软件 [412]。

The third, and current, generation is characterized by the emergence of new algorithms whose cost is only polynomial, allowing for the first time the study of examples with hundreds or thousands of moduli, as well as the construction of large ensembles of compactifications, for example, across the entire Kreuzer-Skarke list. Third-generation approaches to computing intersection numbers, Mori and Kähler cones, and other classical data of Calabi-Yau threefold hypersurfaces are built into version 1.0 of the software CYTools [98]. Third-generation algorithms for computing period integrals and Gopakumar-Vafa invariants through the mirror map appeared in [89], while orientifolding at this level was demonstrated in [24]; both capabilities will be integrated into a future release of CYTools. Another characteristic of the third generation of tools is the application of methods of machine learning, which has led to significant progress, in e.g., the computation of Ricci-flat metrics on compact Calabi-Yau threefolds [385-392]. Machine learning has also been applied to explore other aspects of Calabi-Yau geometry in, e.g., [413- 421]; for a review, see [422].

第三代也就是当前这一代方法的特点是出现了成本仅为多项式级的新算法，首次允许研究包含数百或上千模的例子，还能构建大量紧化集合——例如覆盖整个 Kreuzer-Skarke 列表。计算卡拉比-丘三维超曲面的相交数、Mori 锥、凯勒锥和其他经典数据的第三代方法已内置在软件 CYTools 1.0 版本中 [98]。通过镜像映射计算周期积分和 Gopakumar-Vafa 不变量的第三代算法见文献 [89]，该规模的定向模化工作已在文献 [24] 中完成；这两项功能都将整合到 CYTools 的未来版本中。第三代工具的另一个特点是机器学习方法的应用，这已经带来了重大进展，例如在计算紧卡拉比-丘三维流形上的里奇平坦度量方面 [385-392]。机器学习也已被用于探索卡拉比-丘几何的其他方面，例如文献 [413-421]；综述可参见文献 [422]。

Toric geometry is at the heart of much recent progress in computing topological data of string compactifications. A fine, regular, star triangulation of any of the 473,800,776 reflexive polytopes in the Kreuzer-Skarke list [423] defines a toric variety in which the generic anticanonical hypersurface is a smooth Calabi-Yau threefold. Much of the topology of the hypersurface is inherited from the ambient toric variety, and can be read off from the triangulation. Toric geometry thus provides a combinatorial approach to Calabi-Yau compactifications. Advances in triangulation algorithms [98,424], combined with applications of general results in toric geometry [24, 89], have led to polynomial-time approaches to problems that were inaccessible with general computational algebraic geometry methods.

有锥几何是近年来弦紧化拓扑数据计算领域多数进展的核心。对 Kreuzer-Skarke 列表 [423] 中 473800776 个反身多面体任意一个进行精细、正则、星形三角剖分，就能定义一个有锥 varieties，其中一般反典范超曲面就是光滑卡拉比-丘三维流形。超曲面的大部分拓扑性质都继承自环境有锥 variety，可直接从三角剖分中读出。因此有锥几何为卡拉比-丘紧化提供了一种组合方法。三角剖分算法的进展 [98,424]，结合有锥几何一般性结论的应用 [24, 89]，为以往一般计算代数几何方法无法处理的问题给出了多项式时间解法。

All the classical data, and nearly all the holomorphic data, of a type IIB compactification [66] on a Calabi-Yau threefold hypersurface is accessible by the above methods, but the Pfaffian prefactors of Euclidean D3-brane or gaugino condensate superpotential terms require separate treatment. In toroidal models the full moduli dependence of these factors can be computed on the worldsheet [44]. The dependence on D3-brane positions is well-understood in local models [426], and was recently computed in an orientifold of a compact elliptic Calabi-Yau threefold [46]. A systematic method for computing the moduli dependence of the Pfaffian of any rigid Euclidean Dp-brane was developed in [47]. The overall normalization of the contribution of a rigid Euclidean Dp-branes was fixed in [427], building on a series of recent works treating the normalization of Euclidean Dp-brane amplitudes [428-430].

通过上述方法可以得到 IIB 型紧化 [66] 在卡拉比-丘三维超曲面上的所有经典数据，以及几乎所有全纯数据，但欧几里得 D3 膜或戈奇诺凝聚超势项的 Pfaffian 前置因子需要单独处理。在环面模型中，这些因子的完整模依赖可以在世界面上计算 [44]。D3 膜位置的依赖关系在局域模型中已经得到了充分理解 [426]，近期也已在紧椭圆卡拉比-丘三维流形的定向模中完成计算 [46]。文献 [47] 开发了一种计算任意刚性欧几里得 Dp 膜 Pfaffian 模依赖的系统方法。文献 [427] 基于一系列近期处理欧几里得 Dp 膜振幅归一化的工作 [428-430]，确定了刚性欧几里得 Dp 膜贡献的整体归一化。

Even once the data of an effective theory is in hand, exploring the resulting landscape of vacua is computationally challenging: one often faces exponentially many possible choices of flux, and a scalar potential depending on hundreds of moduli and axions [67]. The JAXVacua framework [439], which deploys modern computational methods including automatic differentiation and just-in-time compilation, marks a significant advance in the computational frontier of moduli stabilization. For recent applications, see [189, 440].

即使已经得到了有效理论的全部数据，探索由此得到的真空景观依然存在计算挑战: 人们通常会面临指数级数量的可能通量选择，以及依赖数百个模和轴子的标量势 [67]。JAXVacua 框架 [439] 运用了包括自动微分和即时编译在内的现代计算方法，标志着模稳定计算领域取得了重大进展。近期应用可参见文献 [189, 440]。

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[66] Related capabilities for F-theory models are provided by FTheoryTools [425].

[66] FTheoryTools[425] 为 F 理论模型提供了相关功能。

[67] It was argued in [431-433] that although the flux landscape is very large, it is finite, but the arguments are not yet conclusive. The construction and enumeration of flux vacua was pursued in, e.g., [197,434-437], and techniques for computation in axion landscapes were developed in [438].

[67] 文献 [431-433] 提出，尽管通量景观规模极大，但它仍是有限的，不过这一论证目前尚无定论。通量真空的构建与计数工作已在诸多研究中展开，例如 [197,434-437]，轴子景观的计算技术也已在文献 [438] 中得到开发。

Conclusions

结论

The past two decades have seen striking advances in moduli stabilization, which is a prerequisite for any attempt to connect string theory to low-energy experiments. In this work we have surveyed the leading approaches toward the still-distant goal of stabilization in cosmologically realistic vacua.

过去二十年来，模稳定领域取得了引人注目的进展，而模稳定是任何将弦论与低能实验联系起来的尝试的先决条件。本文中，我们综述了为在宇宙学真实真空实现模稳定这一遥远目标提出的主流研究方案。

The moduli stabilization mechanisms that we have reviewed invoke a common set of sources that contribute to lifting the moduli spaces found in Calabi-Yau vacuum configurations. These sources include quantized fluxes, D-brane and orientifold configurations, and perturbative and non-perturbative quantum effects. Crucially, these sources are not ad hoc modifications of the theory: they are instead necessary components of general compactifications. Setting the fluxes to zero, rather than to one of the vast number of nonzero values allowed by consistency, is a fine-tuning. Similarly, as in any physical system, quantum effects are not optional. Compactifica-tions in which all such sources are omitted generally preserve more supersymmetry, are easier to analyze, and hence were the first to be understood. This state of affairs is akin to the historical development of cosmology, in which maximally symmetric vacuum spacetimes were understood long before realistic inhomogeneous universes. Likewise, the rich landscape of modern string compactifications is built on the older foundation of moduli spaces of highly supersymmetric solutions.

我们综述的模稳定机制利用一组共同的源来抬升卡拉比-丘真空构型中存在的模空间。这些源包括量子化通量、D 膜与定向投影构型，以及微扰和非微扰量子效应。至关重要的是，这些源并不是对理论的临时修改: 它们恰恰是一般紧化中必不可少的组成部分。将通量设为零，而非自洽性允许的海量非零值之一，本身就是一种精细调谐。同理，正如任何物理系统一样，量子效应并不是可选项。省略所有这类源的紧化通常保留更多超对称，更易于分析，因此也更早被人们理解。这一情况类似于宇宙学的发展历程: 极大对称真空时空早在真实非均匀宇宙被理解之前就已经被研究清楚了。与之类似，现代弦紧化的丰富景观也建立在高超对称解模空间的旧基础之上。

The art of moduli stabilization lies not in finding contributions to the scalar potential for the moduli - indeed, such terms are unavoidable in generic compactifi-cations - but in identifying resulting minima that occur in a region of computational control. The Dine-Seiberg problem indicates that arbitrarily precise control is only guaranteed in the region of moduli space where the scalar potential is a runaway, and so nontrivial vacua most naturally appear in regions of strong coupling [68] .

> 模稳定的精髓不在于寻找模标量势的贡献——事实上，这类项在一般紧化中是不可避免的——而在于识别出出现在计算可控区域的最终极小值。戴恩-塞伯格问题表明，只有当模空间区域中标量势呈 runaway 行为时，才能保证任意精度的控制，因此非平凡真空最自然地出现在强耦合区域 [68] 。

Fortunately, the landscape of topological choices - of quantized fluxes, D-brane configurations, and the underlying compactification topology - is vast enough to allow for solutions that are controllable. The specific moduli stabilization scenarios that we reviewed, such as KKLT and LVS, involve strategic choices that cause minima to appear in computable parameter regimes. The necessary choices are discrete fine-tunings, but they are achievable. For example, the selection of flux quanta giving a small classical flux superpotential in the KKLT scenario leads to a vacuum in which the Calabi-Yau volume is large, and this can be accomplished in actual examples [80].

> 幸运的是，拓扑选择的景观——量子化通量、D 膜构型以及底层紧化拓扑——足够广阔，能够容许可控解存在。我们综述的具体模稳定方案，例如 KKLT 和大体积情景 (LVS)，通过策略性选择让极小值出现在可计算的参数区域中。必要的选择属于离散精细调谐，但仍是可以实现的。例如，在 KKLT 方案中选择通量量子数得到小的经典通量超势，最终会得到卡拉比-丘体积较大的真空，这在实际例子中已经可以实现 [80]。

The task of ensuring that all approximations are under control remains the key technical frontier of moduli stabilization. With each advance in understanding corrections to the four-dimensional EFTs of string compactifications, one can reduce the theory errors in existing constructions, or pursue new constructions that furnish a broader and less fine-tuned class of stabilized vacua. Despite the achievements of recent years, the number of actual solutions that have been constructed is extremely small compared to the astronomical number of compactifications known to exist. Moreover, much of what is now understood applies under particular lampposts, such as the geometric regime in Ricci-flat compactifications. Whole realms of vacua lie undiscovered.

> 确保所有近似都处于可控范围，仍然是模稳定领域关键的技术前沿。随着人们对弦紧化四维有效场论修正的理解不断取得进展，我们可以降低现有构造中的理论误差，也可以开展新的构造，得到范围更广、调谐更少的稳定真空分类。尽管近年来成果颇丰，和已知存在的天文数字般的紧化数量相比，目前实际构造出的解的数量仍然极少。此外，目前人们理解的大部分内容都适用于特定的现有框架下，比如里奇平坦紧化的几何区域。整块的真空领域仍有待探索。

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[68] Nature has been kind to us in furnishing weak couplings at high energies: recall that with the spectrum of the Standard Model, all three gauge couplings are weak at energies close to the Planck scale (the QED Landau pole is at exponentially larger energies). This suggests that perhaps the ultraviolet-complete theory will allow weakly coupled vacua at high energies.

[68] 大自然对我们十分优待，在高能处提供了弱耦合: 回想一下，在标准模型谱中，所有三个规范耦合在接近普朗克能标的能量下都是弱耦合 (QED 朗道极点所在的能量指数级大于普朗克能标)。这表明，紫外完备理论或许确实允许高能处的弱耦合真空存在。

Finally, arguably the most important problem in moduli stabilization is obtaining de Sitter solutions that account for the observed acceleration of the universe. We have seen no meaningful evidence of any fundamental obstacle to the existence of de Sitter vacua. However, much work remains to establish a landscape of de Sitter vacua in totally explicit and precisely controlled compactifications of string theory.

最后，模稳定领域最重要的问题可以说就是得到能够解释观测到的宇宙加速的德西特解。我们目前没有发现任何有意义的证据表明德西特真空的存在存在根本性障碍。不过，要在弦论的完全显式、严格可控的紧化中建立德西特真空景观，仍有大量工作要做。

The new computational techniques reviewed in section ”Computational Advances” may prove crucial for addressing these questions, for systematically constructing more general classes of flux compactifications, and eventually for establishing the structure of cosmological solutions of string theory.

“计算进展”一节中综述的新计算技术，可能对解决这些问题、系统构造更广泛的通量紧化分类，乃至最终建立弦论宇宙学解的结构起到关键作用。

Acknowledgments We would like to thank Carlo Angelantonj and Ignatios Antoniadis, and all the editors of the Handbook on Quantum Gravity, for inviting us to write this review, and for their infinite patience through many delays. We also thank Shamit Kachru for early collaboration on this project. We are grateful to Michele Cicoli, Joe Conlon, Naomi Gendler, Jim Halverson, Manki Kim, Sven Krippendorf, David J.E. Marsh, Raffaele Savelli, Elijah Sheridan, Pramod Shukla, Roberto Valandro, and Timm Wrase for comments on a preliminary draft, and we particularly thank Jakob Moritz and Andreas Schachner for many corrections. The work of L.M. was supported in part by NSF grant PHY-2014071.

致谢我们感谢 Carlo Angelantonj、Ignatios Antoniadis 以及《量子引力手册》的所有编辑，感谢他们邀请我们撰写这篇综述，并且在多次延期中始终保持极大的耐心。我们也感谢 Shamit Kachru 在项目早期的合作。我们感谢 Michele Cicoli、Joe Conlon、Naomi Gendler、Jim Halverson、Manki Kim、Sven Krippendorf、David J.E. Marsh、Raffaele Savelli、Elijah Sheridan、Pramod Shukla、Roberto Valandro 和 Timm Wrase 对初稿提出的意见，特别感谢 Jakob Moritz 和 Andreas Schachner 指出了多处修正。L.M. 的工作部分得到 NSF 项目 PHY-2014071 的支持。

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